EE787
MICROWAVE CIRCUITS, DEVICES AND SYSTEMS

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S-PARAMETERS — INTRODUCTION ①

• WHAT ARE THEY? (SEE APPENDIX A)

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

VOLTAGE, CURRENT — Z - MATRIX

$$b_1 = S_{11} a_1 + S_{12} a_2$$
$$b_2 = S_{21} a_1 + S_{22} a_2$$

WAVES — S - MATRIX

$$V_1 = A V_2 - B I_2$$
$$I_1 = C V_2 - D I_2$$

ABCD MATRIX

$$a_1 = T_{11} b_2 + T_{12} a_2$$
$$b_1 = T_{21} b_2 + T_{22} a_2$$

T - MATRIX

• WHY LEARN S - PARAMETERS?

1) COMPONENTS ARE SPECIFIED BY S - PARAMETERS
2) INSTRUMENTS (NETWORK ANALYZERS) MEASURE S - PARAMETERS
3) THEY ARE USED IN DESIGN

②

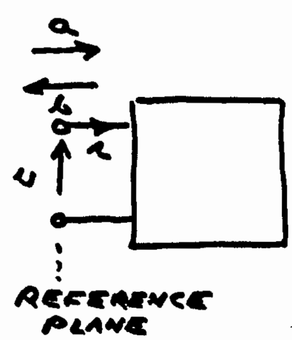
• WHAT ARE WAVES a AND b?

$$a \equiv \frac{V + I Z_0}{2 Z_0^{1/2}} \qquad b \equiv \frac{V - I Z_0}{2 Z_0^{1/2}}$$

UNITS: WATT$^{1/2}$

$Z_0 \equiv$ ARBITRARY NORMALIZING IMPEDENCE

TYPICALLY 50 OR 1 OHM



$$V = (a+b) Z_0^{1/2} \qquad I = (a-b) Z_0^{-1/2}$$

• RELATION TO AVERAGE POWER, $\bar{P}$

IF $v(t) = |V| \cos(\omega t + \varphi) = \mathrm{Re}\, V e^{j\omega t}$

$$\bar{P} = \mathrm{Re}\, \frac{I V^*}{2} = \frac{|a|^2 - |b|^2}{2}$$

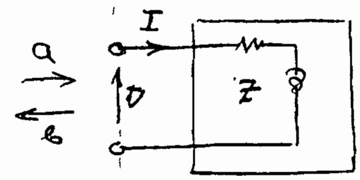
POWER FLOW IN DIRECTION OF I AND a

NOTE: INDEPENDENT OF Z_0

(3)

- PASSIVE ONE-PORT NETWORKS

CONVENTION IS TO USE SYMBOL Γ FOR REFLECTION COEFFICIENT (RATHER THAN S_{11})



WHAT IS RELATION OF Γ TO Z?

REFLECTED WAVE = REFLECTION COEFFICIENT × INCIDENT WAVE

$$b = \Gamma \text{ OR } S \times a$$

USE WAVE RELATIONS TO I, v TO DETERMINE RELATION OF Γ TO Z

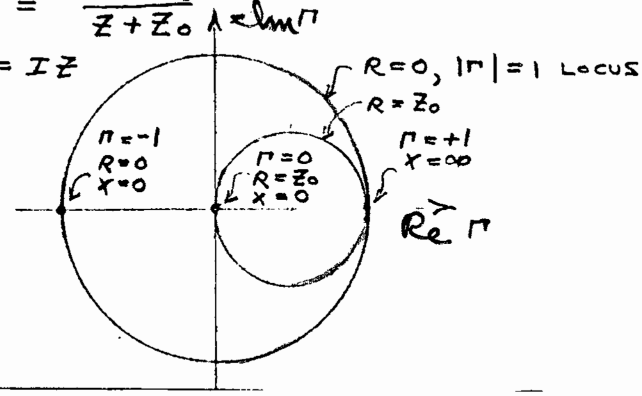
$$\Gamma = \frac{b}{a} = \frac{(v - IZ_0)/2\cancel{Z_0^{1/2}}}{(v + IZ_0)/2\cancel{Z_0^{1/2}}} = \frac{Z - Z_0}{Z + Z_0}$$

$$v = IZ$$

OR SOLVING FOR Z

$$Z = Z_0 \cdot \frac{1 + \Gamma}{1 - \Gamma} = R + jX$$

SMITH CHART IS JUST THE COMPLEX Γ PLANE SHOWING LINES OF CONSTANT R AND X

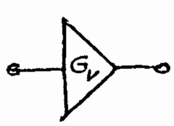


(4)

- S MATRIX EXAMPLES

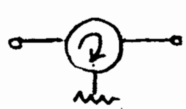
$$\begin{bmatrix} 0 & 0 \\ G_V & 0 \end{bmatrix}$$

MATCHED AMPLIFIER WITH VOLTAGE GAIN, G_V



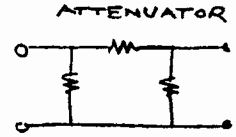
$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

IDEAL ISOLATOR



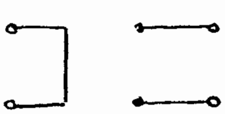
$$\begin{bmatrix} 0 & .707 \\ .707 & 0 \end{bmatrix}$$

3 DB MATCHED ATTENUATOR



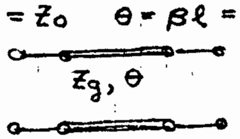
$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

SHORT/OPEN



$$\begin{bmatrix} 0 & e^{-j\theta} \\ e^{-j\theta} & 0 \end{bmatrix}$$

TRANSMISSION LINE WITH $Z_g = Z_0$ $\theta = \beta \ell = 2\pi \ell / \lambda_g$



$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

• CONSTRAINTS ON S MATRIX DUE TO PHYSICAL SYMMETRY OR PROPERTIES OF MATERIALS IN NETWORK

• 2-PORT SYMMETRY

$$S_{11} = S_{22} \qquad S_{12} = S_{21} \qquad \text{OBVIOUS!}$$

• A NETWORK IS RECIPROCAL UNLESS IT CONTAINS:
1) ACTIVE DEVICES SUCH AS TRANSISTORS WHICH REQUIRE A DC BIAS SOURCE AND CAN HAVE POWER GAIN,
OR
2) SPECIAL MATERIALS SUCH AS SOME FERRITES WHICH HAVE TENSOR (RATHER THAN SCALAR) PERMEABILITY, μ, OR PERMATIVITY, ϵ.

FOR A RECIPROCAL NETWORK $S_{ik} = S_{ki}$

$S_{12} = S_{21}$ FOR A TWO PORT EVEN IF IT IS NOT SYMMETRICAL!

• A NETWORK IS LOSSLESS IF IT CONTAINS ONLY DIELECTRICS ($\sigma = 0$) AND CONDUCTORS ($\sigma = \infty$) – NO RESISTANCE MATERIAL OR HEAT DISSAPATION. THE TOTAL AVERAGE POWER ENTERING AN N-PORT NETWORK MUST BE ZERO FOR ANY EXCITATION:

$$\bar{P} = \frac{1}{2}\sum_{k=1}^{N} I_k V_k^* = \frac{1}{2}\sum_{k=1}^{N}\left(|a_k|^2 - |b_k|^2\right) = 0$$

IN MATRIX FORM WITH a AND b AS COLUMN MATRICES:

$$\sum_{k=1}^{N}|a_k|^2 = a^T a^* \qquad \sum_{k=1}^{N}|b_k|^2 = b^T b^* \qquad b] = [S]\, a]$$

$$\bar{P} = a^T(1 - S^* S^T)a^* = 0$$

THUS $S^* S^T = 1$, S IS UNITARY

$S\, S^{\dagger} = 1$

$S^{\dagger} = S^{-1}$

(6)

IF S IS UNITARY $\sum_{j=1}^{N} S_{ij} S_{kj}^* = \delta_{ik} \equiv \begin{cases} 1 & \text{IF } i = k \\ 0 & \text{IF } i \neq k \end{cases}$ FOR ALL i AND k

FOR TWO-PORT CASE ($N = 2$) ABOVE GIVES,

1) $|S_{11}|^2 + |S_{12}|^2 = 1$
2) $|S_{22}|^2 + |S_{21}|^2 = 1$
3) $S_{11} S_{21}^* + S_{12} S_{22}^* = 0$
4) $S_{21} S_{11}^* + S_{22} S_{12}^* = 0$ (THIS IS * OF 3) AND IS SUPERFLUOUS)

IN SUMMARY, FOR A TWO-PORT

IN GENERAL,	8	NUMBERS	ARE	REQUIRED	TO	SPECIFY	S
IF RECIPROCAL,	6	"	"	"	"	"	"
IF SYMMETRICAL,	4	"	"	"	"	"	"
IF RECIPROCAL AND LOSSLESS,	3	"	"	"	"	"	"

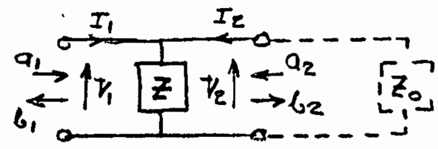
FOR THE LOSSLESS AND RECIPROCAL TWO-PORT, IF 3 PARAMETERS ARE KNOWN, THE REMAINING 5 ARE DETERMINED. FOR EXAMPLE IF $|S_{11}|$, ϕ_{11}, AND ϕ_{22} ARE KNOWN (ϕ_{ik} IS ANGLE OF S_{ik})

$$|S_{22}| = |S_{11}| \qquad |S_{12}| = |S_{21}| = \sqrt{1 - |S_{11}|^2}$$

$$\phi_{12} = \phi_{21} = \frac{\phi_{11} + \phi_{22}}{2} + \frac{\pi}{2} + m\pi$$

(7)

• EXAMPLE — FIND TWO-PORT S MATRIX OF SHUNT Z



FROM SYMMETRY

$$S_{11} = S_{22}$$
$$S_{12} = S_{21}$$

$$S_{11} = \left.\frac{b_1}{a_1}\right]_{a_2=0} \qquad S_{21} = \left.\frac{b_2}{a_1}\right]_{a_2=0}$$

$a_2 = 0$ CAN BE OBTAINED IF LOAD ON PORT 2 HAS ZERO REFLECTION COEFFICIENT, $\Gamma_L = 0$ OR $Z_L = Z_0$

SINCE Z IS A CURRENT-VOLTAGE QUANTITY FIND I_1, V_1, I_2, AND V_2 AND THEN CONVERT TO WAVE VARIABLES

$$V_1 = Z_{in} I_1 = \frac{Z Z_0}{Z + Z_0} I_1 \qquad S_{11} = \frac{b_1}{a_1} = \frac{V_1 - Z_0 I_1}{V_1 + Z_0 I_1} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

$$S_{11} = \frac{\dfrac{Z Z_0}{Z+Z_0} - Z_0}{\dfrac{Z Z_0}{Z+Z_0} + Z_0} = \frac{-Z_0}{2Z + Z_0} = \frac{-1}{2z+1} \qquad \text{WHERE } z = Z/Z_0$$

$$V_2 = V_1 = Z_{in} I_1 \qquad I_2 = -V_2/Z_0 \qquad S_{21} = \frac{b_2}{a_1} = \frac{V_2 - Z_0 I_2}{V_1 + Z_0 I_1} = \frac{Z_{in} I_1 + Z_0 Z_{in} I_1 / Z_0}{(Z_{in} + Z_0) I_1}$$

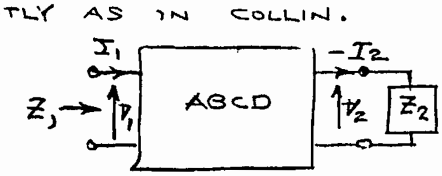
$$S_{21} = \frac{2 Z_{in}}{Z_{in} + Z_0} = \frac{2 Z Z_0}{2 Z Z_0 + Z_0^2} = \frac{2Z}{2Z + Z_0} = \frac{2z}{2z+1}$$

(8)

• ABCD AND T PARAMETERS

ONLY USEFUL FOR TWO-PORT NETWORKS

ABCD DEFINITIONS VARY SLIGHTLY AS IN COLLIN.

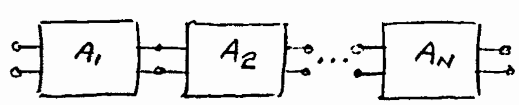


$$V_1 = A V_2 - B I_2$$
$$I_1 = C V_2 - D I_2$$
$$Z_1 = \frac{V_1}{I_1} = \frac{A V_2 - B I_2}{C V_2 - D I_2}$$
$$V_2 = -I_2 Z_2$$
$$Z_1 = \frac{A Z_2 + B}{C Z_2 + D}$$

INPUT IMPEDENCE IS BILINEAR TRANSFORMATION OF LOAD IMPEDENCE. MUCH SIMPLER THAN Z OR Y MATRIX EXPRESSIONS.

CASCADE OF NETWORKS BY MATRIX MULTIPLICATION

$$[A] = [A_1][A_2]\cdots[A_N]$$



T PARAMETERS HAVE SIMILAR PROPERTIES IN TERMS OF REFLECTION COEFFICIENTS

$$a_1 = T_{11} b_2 + T_{12} a_2$$
$$b_1 = T_{21} b_2 + T_{22} a_2$$

ALSO FOR CASCADE OF NETWORKS

$$[T] = [T_1][T_2]\cdots[T_N]$$

$$\Gamma_1 = \frac{b_1}{a_1} = \frac{T_{21} + T_{22}\Gamma_2}{T_{11} + T_{12}\Gamma_2}$$

$$\Gamma_2 = \frac{a_2}{b_2}$$

EQUIVALENT NETWORKS

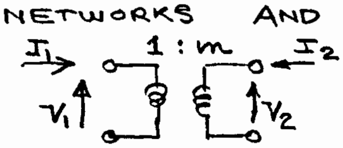
- TWO NETWORKS ARE EQUIVALENT IF THEIR TERMINAL CHARACTERISTICS ARE THE SAME. THE Z, S, ABCD, AND T MATRICES OF ONE NETWORK ARE EQUAL TO CORRESPONDING MATRICES OF THE SECOND NETWORK.

- EQUIVALENCE CAN BE BETWEEN ACTUAL STRUCTURES (I.E. A WAVEGUIDE DEVICE HAVING AN EQUIVALENT COAXIAL DEVICE), BETWEEN CIRCUIT REPRESENTATIONS, OR BETWEEN AN ACTUAL STRUCTURE AND A CIRCUIT.

- A PHYSICAL EQUIVALENT CIRCUIT IS ONE IN WHICH THE CIRCUIT ELEMENTS CAN BE IDENTIFIED WITH PORTIONS OF A PHYSICAL STRUCTURE. IT IS UNIQUE AND IS EQUIVALENT OVER A WIDE RANGE OF FREQUENCY. AN EXAMPLE IS SHOWN ON THE LAST PAGE OF THE NE710 TRANSISTOR DATA SHEET.

- A NON-PHYSICAL EQUIVALENT CIRCUIT IS NOT UNIQUE AND MAY REPRESENT THE DEVICE OVER ONLY A NARROW RANGE OF FREQUENCY. IT IS CHOSEN FOR SIMPLICITY (I.E. 3 ELEMENTS FOR ANY LOSSLESS STRUCTURE) AND IS USEFUL EITHER FOR PERFORMING MEASUREMENTS OR IN PERFORMING A FUNCTION WITH THE DEVICE.

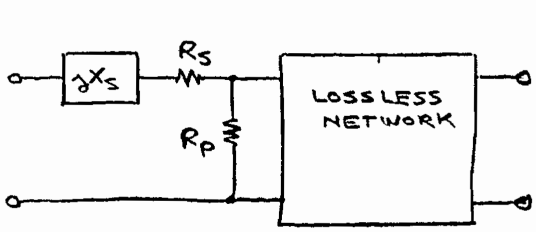
- IDEAL TRANSFORMERS ARE OFTEN USEFUL IN EQUIVALENT NETWORKS AND ARE DEFINED AS:

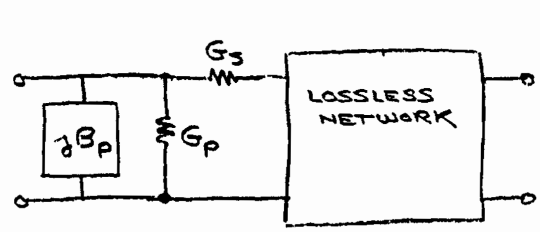


$$V_2 = mV_1 \qquad I_2 = -I_1/m \qquad Z_{in} = Z_\ell / m^2$$

- EIGHT MODELS FOR REPRESENTATION OF LOSSLESS, RECIPROCAL NETWORKS ARE GIVEN IN APPENDIX B. NOTE ALL HAVE 3 PARAMETERS

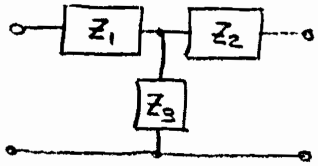
- A LOSSY, RECIPROCAL NETWORK HAS 6 PARAMETERS AND CAN BE REPRESENTED AS A 3 ELEMENT LOSSY NETWORK CASCADE WITH ANY OF THE LOSSLESS NETWORKS AS BELOW:



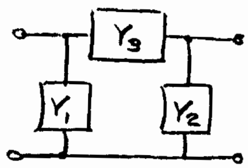


REFERENCE: L.B. FELSEN AND A. OLINER, "DETERMINATION OF EQUIVALENT CIRCUIT PARAMETERS FOR DISSIPATIVE MICROWAVE STRUCTURES", PROC. IRE, FEB 1954, p. 477

T - MODEL



π - MODEL



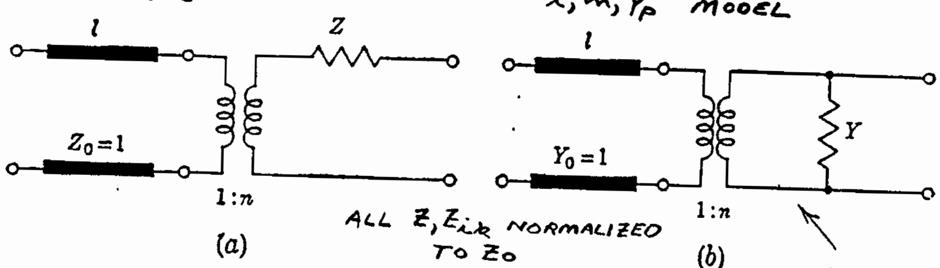
SOME EXAMPLES OF EQUIVALENT NETWORKS

(11)

$Z_{11} = Z_1 + Z_3$ | $Z_1 = Z_{11} - Z_{12}$
$Z_{22} = Z_2 + Z_3$ | $Z_2 = Z_{22} - Z_{12}$
$Z_{12} = Z_3$ | $Z_3 = Z_{12}$

$Y_{11} = Y_1 + Y_3$ | $Y_1 = Y_{11} + Y_{12}$
$Y_{22} = Y_2 + Y_3$ | $Y_2 = Y_{22} + Y_{12}$
$Y_{12} = -Y_3$ | $Y_3 = -Y_{12}$

l, m, Z_S MODEL l, m, Y_P MODEL



ALL Z, Z_{ik} NORMALIZED TO Z_0

FIG. 4·22.

DUAL IS DESCRIBED BY SAME EQUATIONS SUBSTITUTING Y, Y_{ik} FOR Z, Z_{ik}

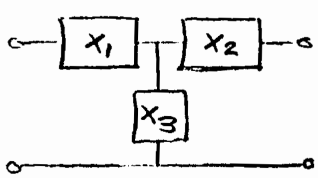
$$Z_{11} = -j\cot \beta l,$$
$$Z_{22} = Z - jn^2 \cot \beta l,$$
$$Z_{12} = -jn\sqrt{\cot^2 \beta l + 1}, \quad = -jn/\sin \beta l$$

$$\beta l = \cot^{-1} jZ_{11},$$
$$n = \frac{+j\, Z_{12}}{\sqrt{1 - Z_{11}^2}},$$
$$Z = Z_{22} + \frac{Z_{11} Z_{12}^2}{1 - Z_{11}^2}.$$

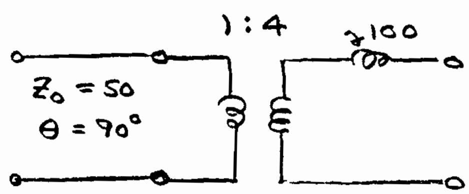
(12)

EXAMPLE APPLICATION OF EQUIVALENT NETWORKS:

SYNTHESIZE A REACTANCE T-NETWORK TO PROVIDE A 50 OHM TO 800 OHM IMPEDENCE STEP-UP AND $+j100$ OF OUTPUT REACTANCE



$\equiv$



$\frac{800}{50} = m^2 \quad m = 4$

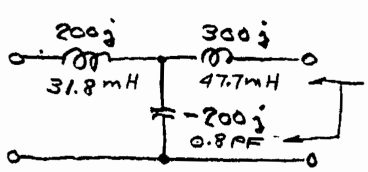
CHOOSE $\theta = 90°$ FOR CONVENIENCE

USING PREVIOUS EXAMPLES

$$Z_{11} = j(X_1 + X_3) = -j\cot\theta = 0$$
$$Z_{22} = j(X_2 + X_3) = Z - jm^2 \cot\theta = j100/50 - 0 = j2$$
$$Z_{12} = jX_3 = -jm\sqrt{\cot^2\theta + 1} = -j4$$

UN NORMALIZE

$X_3 = -200$
$X_2 = 300$
$X_1 = 200$



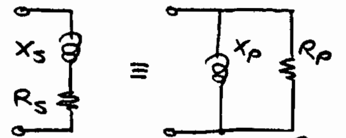
L, C VALUES AT 1 GHz

NOT UNIQUE

COMPUTATION METHODS

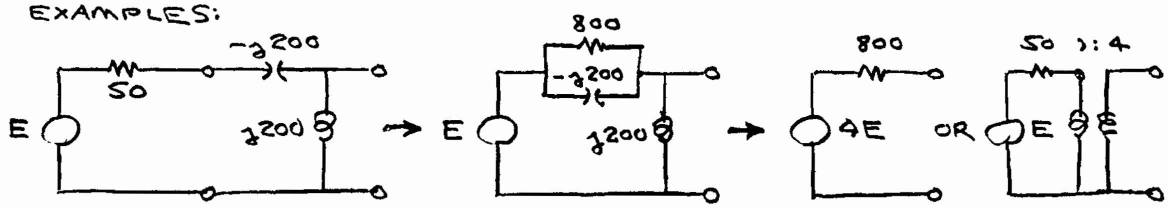
1) SERIES – PARALLEL TRANSFORMATIONS

	EXACT	$Q \gg 1$	$Q \ll 1$
$R_p =$	$R_s(Q^2+1)$	$R_s Q^2$	R_s
$X_p =$	$X_s(1+Q^{-2})$	X_s	X_s/Q^2
$R_s =$	$R_p/(Q^2+1)$	R_p/Q^2	R_p
$X_s =$	$X_p/(1+Q^{-2})$	X_p	$X_p Q^2$



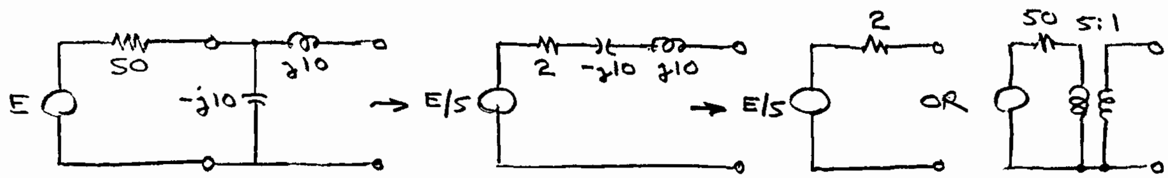
$$Q = \frac{|X_s|}{R_s} \qquad Q = \frac{R_p}{|X_p|}$$

EXAMPLES:



$Q = 4$

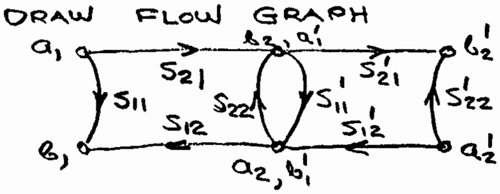
A "Q TRANSFORMER"



2) TOPOLOGICAL METHOD: "NON-TOUCHING LOOP" RULE

EXAMPLE: FIND S''_{11} OF 2 CASCADED S MATRICES

a) DRAW FLOW GRAPH



NOTE:

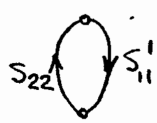
i) VARIABLES (a, b) ARE NODES WITH TWO NODES PER REF PLANE

ii) BRANCHES ARE S PARAMETERS

iii) VARIABLE AT NODE = (BRANCH INTO NODE) × (VARIABLE AT INPUT OF BRANCH)

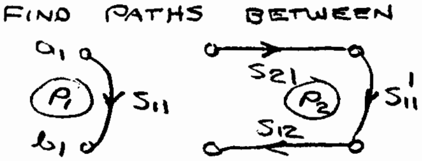
i.e. $b_1 = S_{11} a_1 + S_{12} a_2$

b) FIND FIRST ORDER LOOPS, $L(1)$



c) FIND SECOND ORDER LOOPS AS PRODUCTS OF NON-TOUCHING FIRST ORDER LOOPS – NONE IN THIS CASE

d) FIND PATHS BETWEEN VARIABLES YOU WISH TO RELATE



e) APPLY RULE

$$S''_{11} = \frac{b_1}{a_1} = \frac{\overbrace{S_{11}}^{P_1}\overbrace{(1 - S_{22}S'_{11})}^{\Sigma \text{ OF } L(1)\text{'s NOT TOUCHING } P_1} + \overbrace{S_{21}S'_{11}S_{12}}^{P_2}(1-0)}{\underbrace{1 - S_{22}S'_{11}}_{\Sigma \text{ OF ALL } L(1)\text{'s}}}$$

SEE NOTES PAGE FOR FULL DEFINITION

14A

FROM MICROWAVE THEORY AND APPLIC. S.F. ADAM (1969) p. 87

3.2.4 ANALYTICAL APPROACH TO RESOLVE FLOW GRAPHS

The "nontouching-loop" rule is an analytical method for solving a flow graph. This technique is fast, but it is very easy to forget and miss a few loops. Although the topographical approach of resolving flow graphs takes longer to do, it is very easy to remember. A few basic definitions have to be understood before the nontouching-loop rule can be learned.

A "path" is a series of branches followed in sequence and in the same direction in such a manner that no node is touched more than once. The "value of the path" is the product of the coefficients of the branches encountered en route.

Figure 3.2-18 shows the flow graph of a two-port network driven with a signal source and terminated with a load. One path goes from the generator to node b_2; its value is s_{21}. There are two paths from the generator to node b_1. The values of these paths are s_{11} and $s_{21}\Gamma_L s_{12}$.

If a path starts and finishes in the same node, it is called a "loop," rather than a path. A "first-order loop" is a path coming to a closure with no node passed more than once. The value of the loop is calculated as the value of the path, or the product of the value of all branches encountered en route.

A "second-order loop" is defined as two first-order loops not touching each other at any node. The value of a second-order loop is the product of the values of the two first-order loops. Third- and higher-order loops are three or more first-order loops not touching each other at any point. Their values are calculated in the same manner as described above for the second-order loop, that is, by multiplying the coefficients of branches encountered.

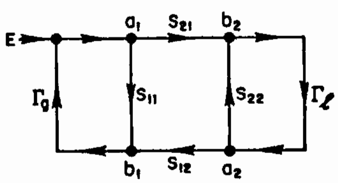


Fig. 3.2-18. Flow graph of a two-port network with a signal source and a load.

For example, in Fig. 3.2-18, there are three first-order loops ($s_{11}\Gamma_g$, $s_{22}\Gamma_L$, and $\Gamma_g s_{21}\Gamma_L s_{12}$) and one second-order loop ($\Gamma_g s_{11} s_{22}\Gamma_L$).

The nontouching-loop rule[8,9] can be applied to solve any flow graph. The equation in symbolic form is

$$T = \frac{\begin{aligned} &P_1[1 - \Sigma L(1)^{(1)} + \Sigma L(2)^{(1)} - \Sigma L(3)^{(1)} + \ldots] \\ &\quad + P_2[1 - \Sigma L(1)^{(2)} + \Sigma L(2)^{(2)} - \ldots] \\ &\quad + P_3[1 - \Sigma L(1)^{(3)} + \ldots] + P_4(1 - \ldots) + \end{aligned}}{1 - \Sigma L(1) + \Sigma L(2) - \Sigma L(3) + \ldots}$$

where $\Sigma L(1)$ stands for the sum of all first-order loops, $\Sigma L(2)$ is the sum of all second-order loops, and so on; P_1, P_2, P_3, etc., stand for the values of all paths that can be followed from the independent variable, in most cases the generator, to the node whose value is desired; $\Sigma L(1)^{(1)}$ denotes the sum of those first-order loops which do not touch the path of P_1 at any node; $\Sigma L(2)^{(1)}$ denotes then the sum of those second-order loops which do not touch the path of P_1 at any point; $\Sigma L(1)^{(2)}$ consequently denotes the sum of those first-order loops which do not touch the path of P_2 at any point. Each path is multiplied by the factor in parentheses which involves all the loops of all orders that the path does not touch. T represents the ratio of the dependent variable in question and the independent variable.

The example shown in Fig. 3.2-18 can be calculated for two dependent variables. One is the reflection coefficient of the two-port network b_1/a_1, and the second is the transmission coefficient b_2/E. In the first case, when b_1/a_1 is to be found, the generator is not involved, so it should be neglected. The solution is

$$\frac{b_1}{a_1} = \frac{s_{11}(1 - s_{22}\Gamma_L) + s_{21}\Gamma_L s_{12}}{1 - s_{22}\Gamma_L}$$

s_{11} is the first path, P_1, which has to be multiplied with $1 - \Sigma L(1)^{(1)}$. $s_{22}\Gamma_L$ is

[8] Lorens, C. S., "A Proof of the Nonintersecting Loop Rule for the Solution of Linear Equations by Flow Graphs," Res. Lab. of Electronics, M.I.T., Cambridge, Mass., *Quarterly Progress Report*, January 1956, pp. 97–102.

[9] Happ, W. W., "Lecture Notes on Signal Flow Graphs," from *Analysis of Transistor Circuits*, Extension Course, University of California, Catalog 834AB.

Nontouching-Loop Rule

$$T = \frac{P_1(1 - \Sigma L(1)^{(1)} + \Sigma L(2)^{(1)} - \Sigma L(3)^{(1)} + \ldots) + P_2(1 - \Sigma L(1)^{(2)} + \Sigma L(2)^{(2)} - \ldots) + P_3(1 - \Sigma L(1)^{(3)} + \ldots) + \ldots}{1 - \Sigma L(1) + \Sigma L(2) - \Sigma L(3) + \ldots}$$

$\Sigma L(1)$ Sum of all first-order loops
$\Sigma L(2)$ Sum of all second-order loops
P_1, P_2, P_3 Values of paths corresponding to indices
$\Sigma L(1)^{(1)}$ Sum of those first-order loops which do not touch P_1
$\Sigma L(2)^{(1)}$ Sum of those second-order loops which do not touch P_1
$\Sigma L(n)^{(m)}$ Sum of those n-order loops which do not touch P_m path
T Ratio of dependent variable in question and independent variable

3) MICROWAVE CAD SURVEY

15

TASK	FARANT	COMPACT OR TOUCHSTONE	OTHER PROGS	COMMENT
INPUT Z CALCS, COMPUTERIZED SMITH CHART	Y	Y	LADDER	WRITE YOUR OWN!
TWO-PORT CIRCUIT ANALYSIS	Y	Y	ALMOND	HANDLES MOST TOPOLOGIES
NODAL CIRCUIT ANALYSIS	N	Y	MIC SPICE	ALL TOPOLOGY
CIRCUIT OPTIMIZATION	Y	Y	—	REQUIRES MUCH COMPUTATION
MATRIX PARAMETER CONVERSION	Y	S	—	
NOISE ANALYSIS	Y	S	—	FOR LOW-NOISE DESIGNS
PHYSICAL TO ELECTRICAL PARAMETER CONVERSION	N	Y	MSTRIP, MLINE	EQUIVALENT CIRCUITS FOR STRIP TRANSMISSION LINES AND DISCONTINUITIES
FILTER SYNTHESIS	N	N	FILSYN	
CIRCUIT LAYOUT, 2-D DRAWING	N	N	MICAD, AUTOCAD	GENERATE ARTWORK FOR MICROSTRIP
NON-LINEAR ANALYSIS	N	N	SPICE, GISS MIX	FOR MIXERS, OSCILLATORS, AND HIGH POWER AMPS
FIELD THEORY SOLUTION	N	N	ANSOFT	FOR ODD GEOMETRIES

CAD EXAMPLES:

16

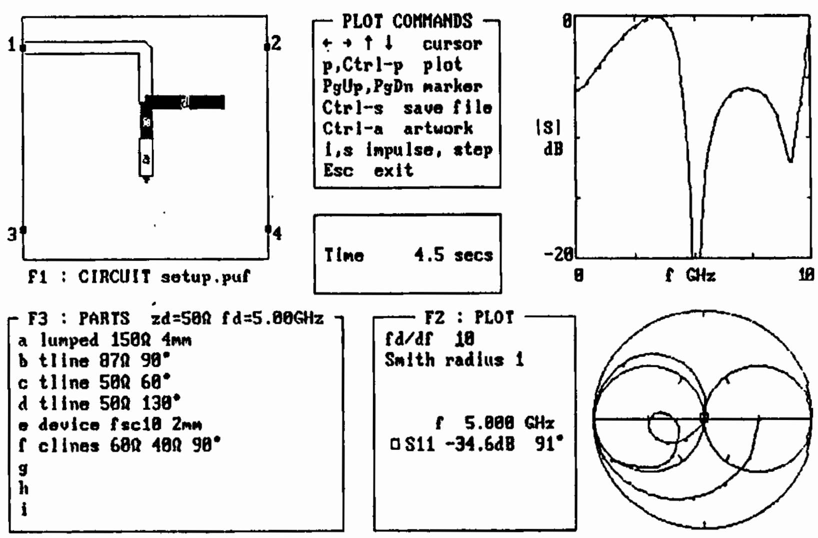
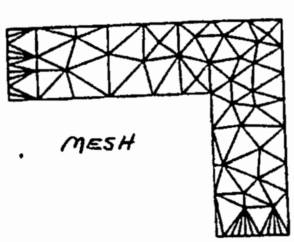


Figure 1.5 Shunt-stub matching.

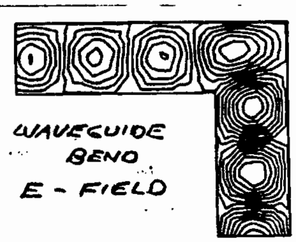
PUFF (CALTECH)

MICROSTRIP LAYOUT AND ANALYSIS

ALL ON CRT AT ONE TIME!



MESH



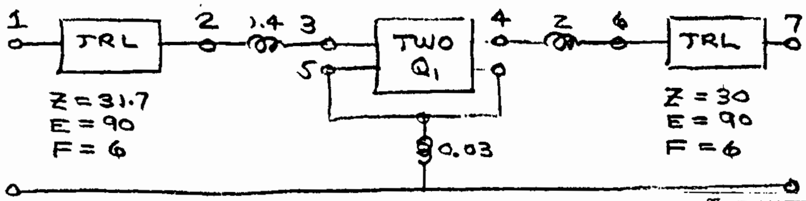
WAVEGUIDE BEND E-FIELD

ANSOFT MAXWELL

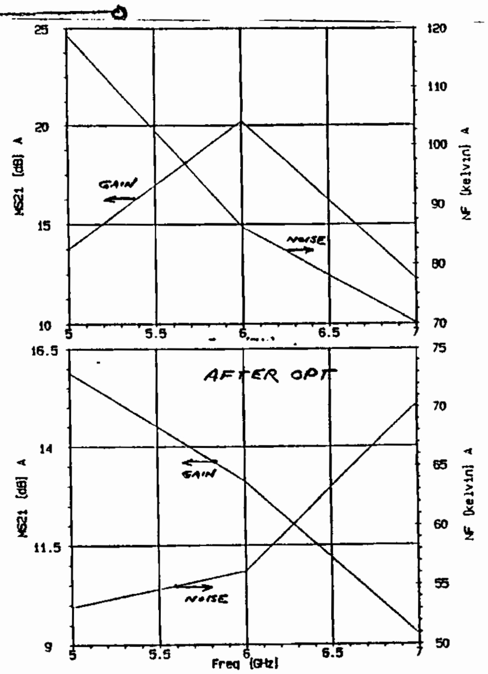
FINITE ELEMENT ELECTROMAGNETIC FIELD ANALYSIS

CAD EXAMPLE – COMPACT – NODAL ANALYSIS AND OPTIMIZATION

17



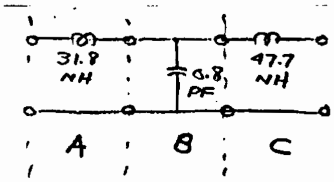

```
SUPER COMPACT PC   07/24/87   17:20:39   File: noi2.ckt
*    Dr s weinreb  test            87 7 24
*NOI
NOI
   TRL 1 2        Z=?31.67?  E=90    F=6GHZ
   IND 2 3        L=?1.4077NH?
   TWO 3 4 5      Q1
   IND 5 0        L=?.02652NH?
   IND 4 6        L=?1.9738NH?
   TRL 6 7        Z=30   E=90   F=6GHZ
A:   2POR 1 7
*B:   2POR 7 1
END
*
FREQ
   STEP   5GHZ   7GHZ   1GHZ
END
*
OUT
  PRI A S
END
*
OPT
   A   NF=.5 W=10   MS21=12   W=2
END
*
DATA
Q1: S
  5GHZ  .820    -65      3.811  126     .088   54     .647    -32
  6GHZ  .778    -74      3.503  118     .094   50     .614    -36
  7GHZ  .742    -82      3.224  112     .101   47     .585    -40
NOI   NC
  5GHZ .629   .718    44    1.428
  6GHZ .744   .686    53    1.518
  7GHZ .855   .657    61    1.614
END
```



• FARANT CAD EXAMPLE

18

CIRCUIT



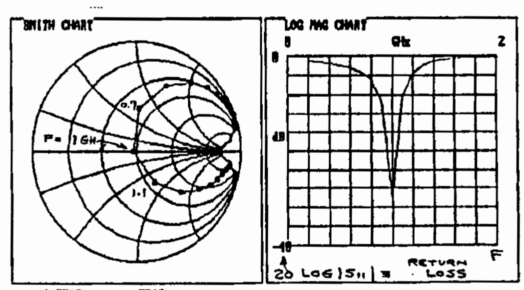
TRUE BASIC

```
LIBRARY "{F:}BLIB.TRC", "{F:}PLIB.TRC", "{F:}COMLIB.TRC"

DECLARE PUBLIC Zo, F, NOGO, ICOUNT

OPEN #1: NAME "{F:}SCHART", CREATE NEWOLD, RECSIZE 40000!
RESET #1: BEGIN                                     !PICT
READ #1: PICTURE$                                   !SMIT
RESET #1: END                                       !DISK

CALL SETUP(#3,#4,#5)                   !SET UP REGIONS DESIG

DO

CALL SMITH(#3,PICTURE$)                  !PRINT CHART IN REG
CALL PLOG(#4)                            !PRINT CHART IN REGI

DIM A(6,4), B(6,4),C(6,4),Dat(51,18)
MAT A = ZER
MAT B = ZER
MAT C = ZER
MAT Dat = ZER
!   Main loop
FOR F = 0.20 TO 2 STEP 0.1
  LET ICOUNT = ICOUNT + 1
  CALL RLC(A,"S",0,31.83,0,"S",0)
  CALL RLC(B,"S",0,0,.796,"P",0)
  CALL RLC(C,"S",800,47.74,1.592,"S",0)
  CALL CAS(A,B)
  CALL CAS(A,C)
  CALL MTRANS(A,4)
  CALL SPLOT(A(1,1),A(1,2),XOLD,YOLD,#3)
  CALL LPLOT(A(1,1),A(1,2),XOLD,YOLD,FOLD,#4)
  CALL DTEXT(A(1,1),A(1,2),#5)
  LET XOLD = A(1,1)
  LET YOLD = A(1,2)
  LET FOLD = F
NEXT F
```

CIRCUIT DESCR.

OUTPUT

OUTPUT

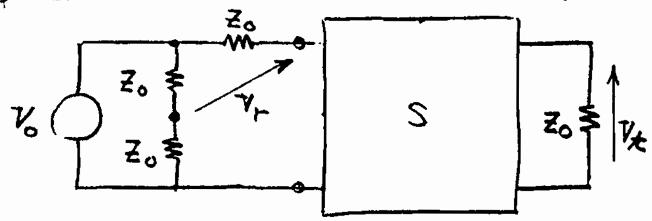


1.5000	-	.5818
1.6000	-	.3696
1.7000	-	.2464
1.8000	-	.1786
1.9000	-	.1217
2.0000	-	.0889

GHZ   20 LOG |S11|

MEASUREMENT TECHNIQUES

1) REFLECTOMETER BRIDGE (MFG BY WILTRON, HP, AND OTHERS)

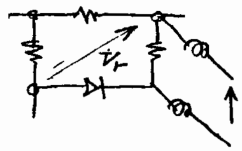


FROM SIMPLE CIRCUIT ANALYSIS,

$$\frac{V_r}{V_o} = \frac{z_{in}-1}{2(z_{in}+1)} = \frac{\Gamma_{in}}{2} = \frac{S_{11}}{2}$$

WHERE $z_{in} = \dfrac{Z_{in}}{Z_o}$

MAGNITUDE AND PHASE OF V_r/V_o COULD BE MEASURED WITH A VECTOR VOLTMETER. USUALLY ONLY MAGNITUDE IS MEASURED WITH A DIODE DETECTOR ACROSS V_r

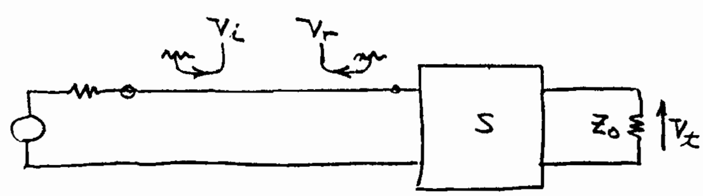


$\uparrow V_{oc} \propto |V_r|^2 \propto |S_{11}|^2$

THUS A SIMPLE BROADBAND DEVICE TO MEASURE THE MAGNITUDE OF S_{11} OR S_{22} IS ACHIEVED. THE MAGNITUDE OF S_{21} OR S_{12} CAN BE MEASURED WITH A DIODE DETECTOR ACROSS V_t. A DEVICE ENCOMPASSING THE BRIDGE, DETECTOR, AND A LOGARITHMIC DISPLAY IS CALLED A <u>SCALAR NETWORK ANALYZER</u>.

---

2) REFLECTOMETER WITH <u>DIRECTIONAL COUPLERS</u>



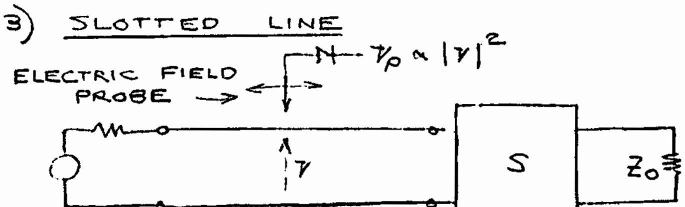
$V_i \propto a_1$ *INCIDENT*

$V_r \propto b_1$ *REFLECTED*

$V_t \propto b_2$ *TRANSMITTED*

DIRECTIONAL COUPLERS COUPLE OUT THE a AND b WAVES AND AGAIN, A VECTOR VOLTMETER CAN GIVE S_{11} AND S_{21} OR DIODE DETECTORS GIVE $|S_{11}|$ AND $|S_{21}|$.

3) <u>SLOTTED LINE</u>



$V = (a+b)\,Z_o^{1/2}$

AS PROBE IS MOVED

$V_{max} = (|a| + |b|)\,Z_o^{1/2}$

$V_{min} = (|a| - |b|)\,Z_o$

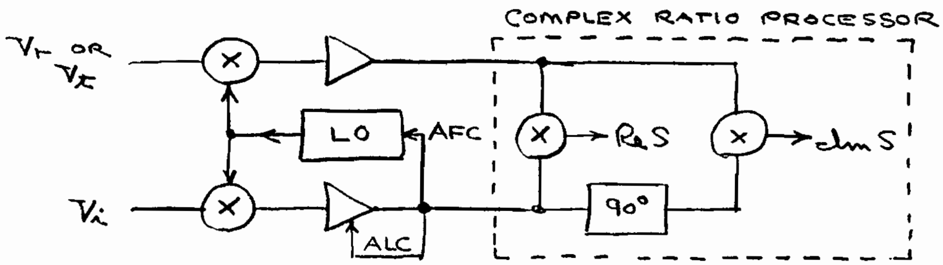
$$\frac{|V|_{max}}{|V|_{min}} \equiv VSWR = \frac{|a|+|b|}{|a|-|b|} = \frac{1+|\Gamma|}{1-|\Gamma|} \quad \text{OR} \quad |\Gamma| = \frac{VSWR-1}{VSWR+1}$$

AT POSITION OF MINIMUM $\angle\Gamma \equiv \phi = \angle b - \angle a = \pi$

THUS BOTH A SLOTTED LINE DETERMINES BOTH MAGNITUDE AND PHASE OF A REFLECTION COEFFICIENT

4) VECTOR NETWORK ANALYZER

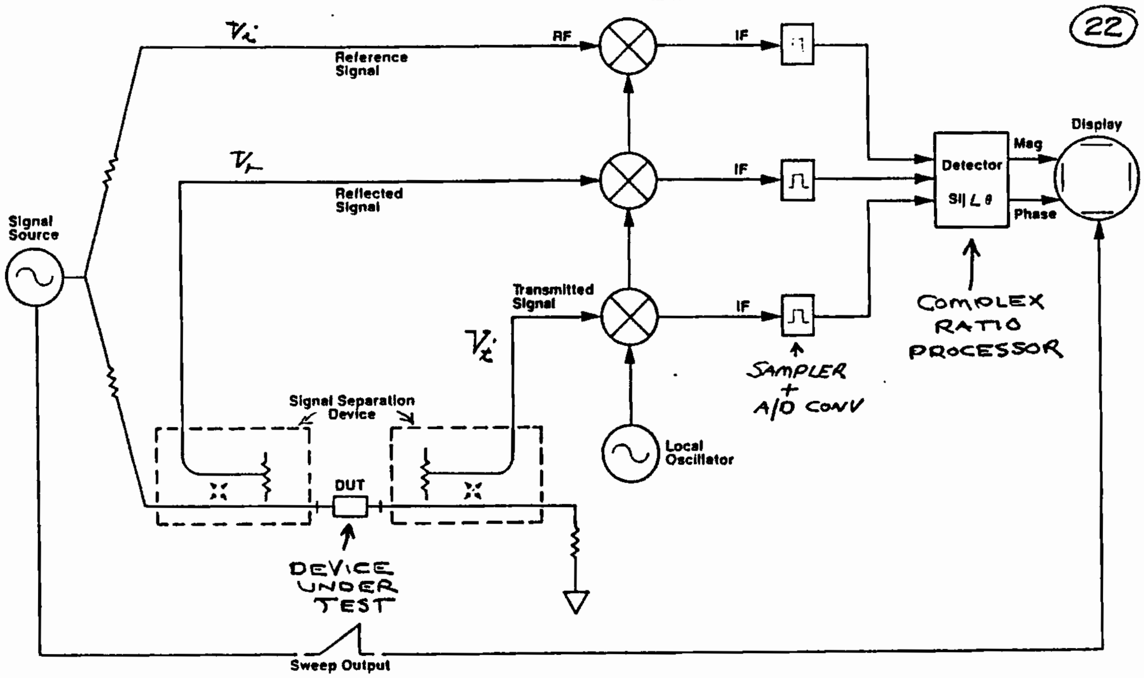
NOTE THAT THE INCIDENT AND REFLECTED VOLTAGES, V_i AND V_r PRODUCED BY THE DIRECTIONAL COUPLERS IN 2) CAN BE MIXED (MULTIPLIED) WITH A COMMON LOCAL OSCILLATOR TO PRODUCE LOW FREQUENCY REPLICAS WITH THE SAME COMPLEX RATIO, $V_r/V_i = S_{11}$



$$V_r \cos(\omega_r t+\phi_r) \times V_L \cos(\omega_\ell t+\phi_\ell) = \frac{V_r V_\ell}{2} \cos\left[(\omega_r-\omega_\ell)t+\phi_r-\phi_\ell\right]$$
$$+ (\omega_r+\omega_\ell) \text{ TERM}$$

A VECTOR ANALYZER IS CALIBRATEABLE. THAT IS, ALMOST ALL ERRORS CAN BE REMOVE BY REPLACING THE UNKNOWN NETWORK WITH 3 SIMPLE BUT KNOWN NETWORKS.

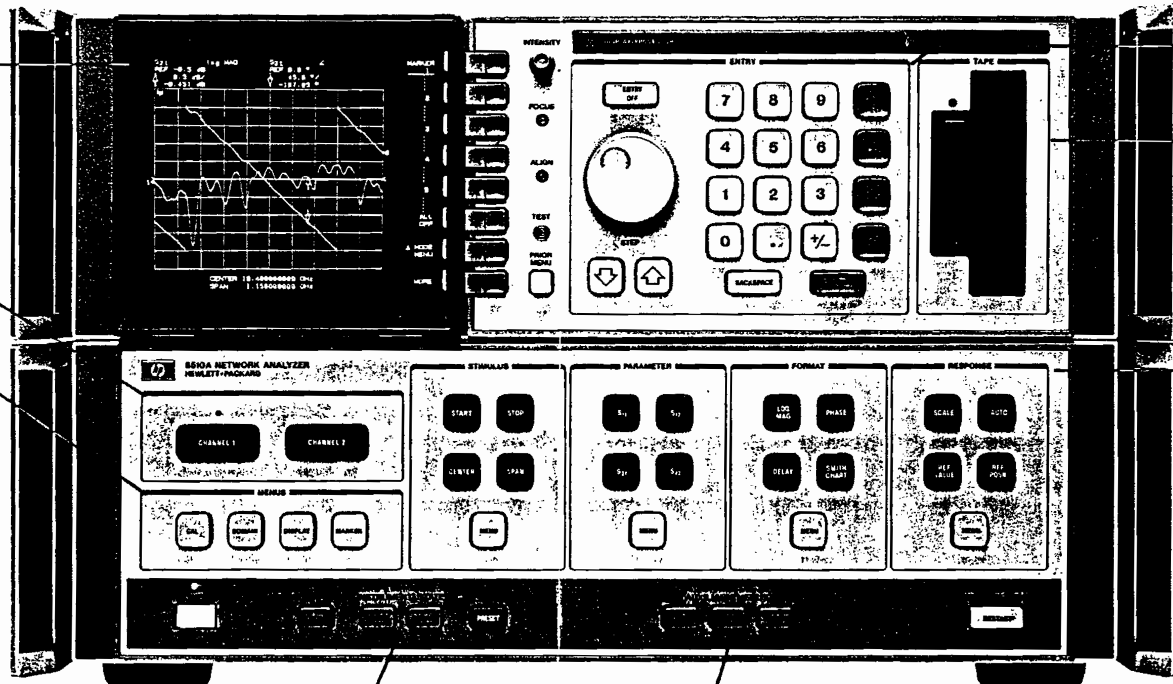
A VECTOR ANALYZER CAN BE REALIZED WITHOUT MIXING IN A CONFIGURATION KNOWN AS A <u>SIX-PORT ANALYZER</u> WHICH IS ILLUSTRATED IN A HOMEWORK PROBLEM



A MORE COMPLETE DIAGRAM OF A VECTOR NETWORK ANALYZER IS SHOWN ABOVE. A VIEW GRAPH OF A COMMERCIAL INSTRUMENT WILL BE SHOWN IN CLASS.

Key Features for Versatile Network Analysis...

HP 8510 NETWORK ANALYZER



Digital Display
Annotation of the current Stimulus frequency limits, the Parameter being measured, its Format, and the Response scale per division and reference line value. Also, the current soft key menu selections are shown.

Two Independent Channels

Menus
The Soft Key Menu structure provides for additional functions and also guides the operator through prompts, such as those actions necessary for calibration.

Cal
Versatile calibration allows for APC-7,® 3.5 mm & other connectors; set up your own connector type (up to 8 different waveguide or coaxial types); then store and retrieve up to 8 different, complete calibrations.

Domain
When you include the Time Domain option with your system, you enable transformations from frequency domain data (magnitude and phase vs. frequency) to the time domain response. Now, in a network analyzer, you can see the effect of impedance discontinuities in time or distance.

Display
Select single channel, 2 channel overlay, or 2 channel split display modes. Further, you can store measured data in one of four memories for normalization and subsequently perform trace mathematics (+, −, ×, ÷) on a selected memory with the displayed data.

Marker
Five fully annotated markers; Δ MODE allows relative readings to any marker. Marker set to Maximum/Minimum.

Instrument State
Save and recall 5 Instrument States and Preset to a known, invariant state.

Auxiliary Menus

Copy
Automatic hard copy output to HP-IB plotter dependent on your pen selection and plot format.

Tape
Store and retrieve additional measured data, calibration data, and instrument states.

System
Provides HP-IB and 8510 System Bus addresses, service functions, the means for generating a CRT title, and other system information.

Entry
Data Entry for all functions is quickly accomplished with Knob, Step Keys, or Keypad.

Tape
Tape Drive provides for additional storage and retrieval of measured data, calibration data, and instrument states.

Principal analyzer functions are divided into four selection groups, each with its Menu key to access other useful and relevant functions.

Stimulus
Easy, direct control of the HP 8350B Sweep Oscillator or the HP 8340A Synthesized Sweeper. Major keys let you set frequencies, and MENU lets you set source power, sweep time, and other related functions.

Parameter
Select the parameter to be measured. With the RF applied to port 1, you select S_{11} for reflection (return loss) and S_{21} for transmission (insertion loss or gain). Likewise, with RF applied to port 2, you select S_{22} for reflection and S_{12} for transmission. Control of the test set, depending on the parameter, is enabled automatically. MENU gives you access to other parameters.

Format
Place measured parameter data in desired format: logarithmic (dB) magnitude, phase, group delay, and Smith Chart. SWR, linear magnitude, R + jX impedance, and others are selected using the MENU.

Response
Set the Scale per division, Reference Level, or Reference Position, or let the AUTO function automatically place all the measured data on the display. MENU functions include averaging, smoothing, and an electronic line stretcher.

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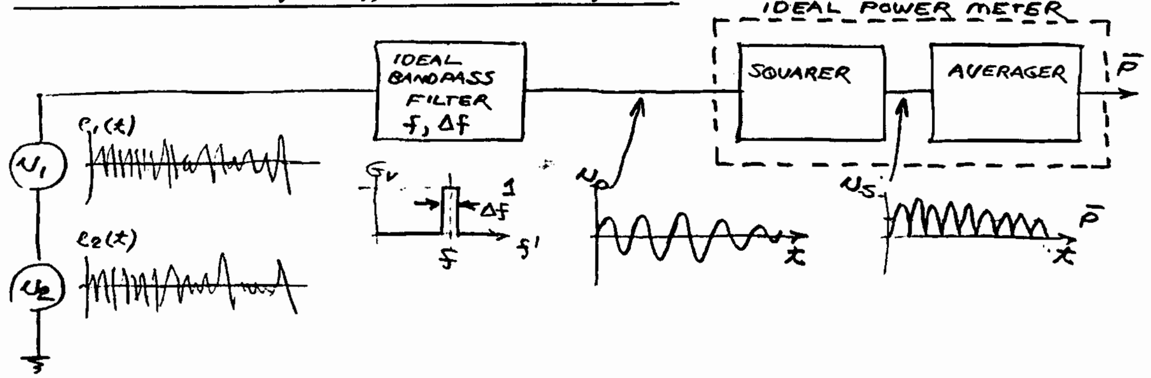
22A

NOISE VOLTAGE, $\mathcal{V}(f)$, AND POWER, $P(f)$



FIRST, CONSIDER $v_2 = 0$

THE POWER SPECTRUM, $P_1(f)$, OF $e_1(t)$ IS DEFINED AS:

$$P_1(f) \equiv \lim_{\Delta f \to 0} \frac{\bar{P}}{\Delta f}$$ WHERE f IS THE FILTER CENTER FREQUENCY AND Δf IS THE BANDWIDTH

THE COMPLEX, RMS NOISE VOLTAGE AMPLITUDE, $\mathcal{V}_1(f)$, IS RELATED TO $P_1(f)$ AND $v_1(t)$ BY:

$$\frac{|\mathcal{V}_1(f)|^2}{R} = P_1(f) \qquad \mathcal{V}_1(f) = \frac{1}{\sqrt{2}} \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} v_1(t) e^{-j\omega t} dt$$

$\equiv \frac{1}{\sqrt{2}} \times$ FOURIER COMPONENT OF $v_1(t)$ AT FREQUENCY f

$\mathcal{V}_1$ IS A RANDOM VARIABLE $\quad \overline{\mathcal{V}_1} = 0 \quad \overline{\mathcal{V}_1^2} \neq 0 \quad \sqrt{\overline{\mathcal{V}_1^2}} = \mathcal{V}_{1\,RMS}$

SOMETIMES WRITTEN $\mathcal{V}_1$

COMPLEX NOISE AMPLITUDES CAN BE TREATED IN MOST CASES JUST AS COMPLEX SINUSOIDAL AMPLITUDES, I.E.

$$\mathcal{V}_1 = Z I_1, \quad \mathcal{V}_T = \mathcal{V}_1 + \mathcal{V}_2, \quad a_1 = \frac{\mathcal{V}_1 + Z_0 I_1}{2\sqrt{Z_0}} \equiv \text{NOISE WAVE AMPLITUDE}$$

(THE (f) ARGUMENT IS UNDERSTOOD)

AN EXCEPTION IS WHEN A SUM OF VARIABLES IS SQUARED THE RESULT DEPENDS ON WHETHER THE VARIABLES ARE CORRELATED AS DESCRIBED BY CORRELATION COEFFICIENT, ρ

$$\rho_{12} \equiv \frac{\overline{\mathcal{V}_1 \mathcal{V}_2^*}}{|\mathcal{V}_1||\mathcal{V}_2|} = |\rho_{12}| e^{j\varphi_{12}} \qquad |\rho_{12}| \leq 1$$

THUS THE TOTAL POWER IN THE SUM OF TWO VOLTAGES, $\mathcal{V}_1$ AND $\mathcal{V}_2$ IS:

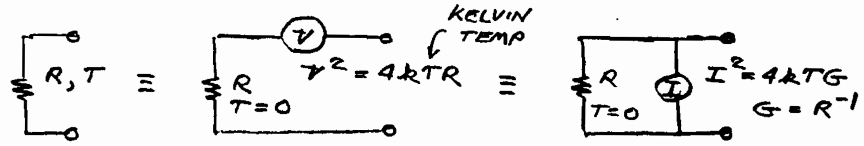
$$P_T = \frac{\overline{|\mathcal{V}_1 + \mathcal{V}_2|^2}}{R} = \frac{\overline{|\mathcal{V}_1|^2}}{R} + \frac{\overline{|\mathcal{V}_2|^2}}{R} + 2\,\mathrm{Re}\,|\overline{\mathcal{V}_1 \mathcal{V}_2^*}|$$

$$= \frac{\overline{|\mathcal{V}_1^2|}}{R} + \frac{\overline{|\mathcal{V}_2|^2}}{R} + 2|\mathcal{V}_1||\mathcal{V}_2|\,\mathrm{Re}\,\rho_{12}$$

$$= \frac{\overline{|\mathcal{V}_1|^2}}{R} + \frac{\overline{|\mathcal{V}_2|^2}}{R}$$ IF V_1 AND V_2 ARE UNCORRELATED, $\rho_{12} = 0$

$$= \frac{\overline{[|\mathcal{V}_1| + |\mathcal{V}_2|]^2}}{R}$$ IF V_1 AND V_2 ARE PERFECTLY CORRELATED, $\rho_{12} = 1$

THERMAL NOISE – JOHNSON NOISE OF A RESISTOR



v, I INDEPENDENT OF f
"WHITE" NOISE, FLAT SPECTRUM

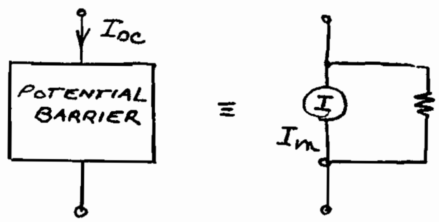
$$\text{AVAILABLE POWER} = \frac{V_{oc}^2}{4R} = \frac{I_{sc}^2 R}{4} = kT = 4\times10^{-21}\ \text{WATTS/HZ}\quad \text{FOR } T = T_0 \equiv 290K$$

$$k = 1.38\times10^{-23}\ \text{WATTS/HZ}\cdot K$$

$V_{oc} = 0.13\sqrt{R}\ \text{NV}/\sqrt{\text{HZ}}$ FOR $T = 290K$ $\qquad I_{sc} = \frac{180}{\sqrt{R}}\ \text{PA}/\sqrt{\text{HZ}}$

$kT_0 = -174\ \text{DBM/HZ} = -114\ \text{DBM/MHZ}$

SHOT NOISE



$$R = \frac{dv}{di} = \frac{1}{\frac{di}{dv}}$$

$I_n^2 = 2qI_{DC}$ $\quad q = 1.6\times10^{-19}$ COULOMB

I_{DC} AMPS	I_n PA/$\sqrt{\text{HZ}}$
1	565
10^{-3}	18
10^{-6}	0.6

EXAMPLE: SCHOTTKY DIODE $\quad i = I_0(e^{qV/kT} - 1)$ $\qquad \frac{di}{dv} = \frac{q}{kT}e^{qV/kT} = R^{-1}$

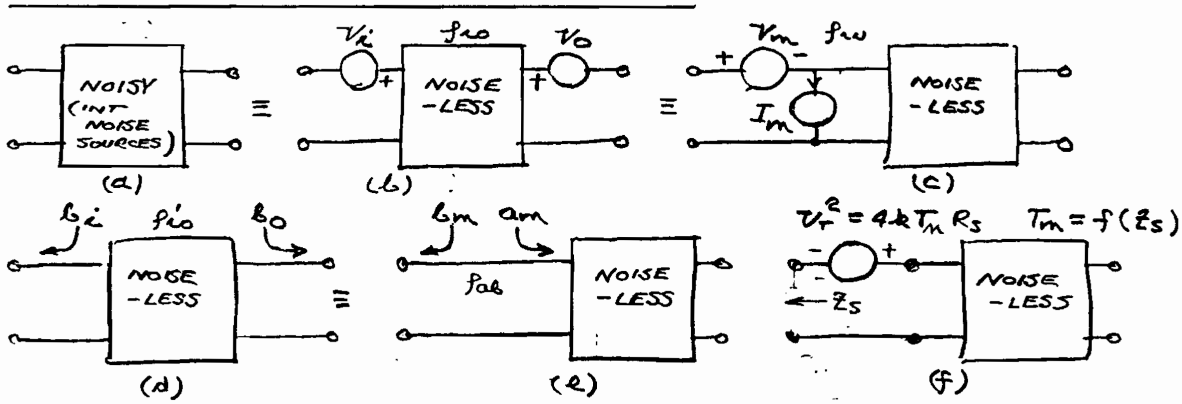
$$\text{AVAILABLE POWER} = \frac{I_n^2 R}{4} = \frac{2qi\cdot kT}{4\ qi} = \frac{kT}{2}$$

$\sim \frac{qi}{kT}$ $\quad i \gg I_0$

@ $i = 0.5\ \text{mA}$
$R = 50$ OHMS

NOISE TEMPERATURE OF BIASED DIODE (EQUIVALENT RESISTOR TEMPERATURE) $= \frac{1}{2}$ PHYSICAL TEMPERATURE

REPRESENTATION OF NOISY NETWORKS



SIX METHODS OF REPRESENTING A NOISY NETWORK
ALL EXCEPT (f) REPRESENT NOISE AT BOTH PORTS
(c) AND (f) ARE MOST COMMON
WHAT IS THE BASIS OF THESE MODELS? – CONSIDER (b)

$$V_1 = Z_{11}I_1 + Z_{12}I_2 + V_i$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2 + V_o$$

THEVENIN THEOREM APPLIED TO AN ACTIVE NETWORK
V_i AND V_o HAVE CORRELATION COEFF $= \rho_{io}$

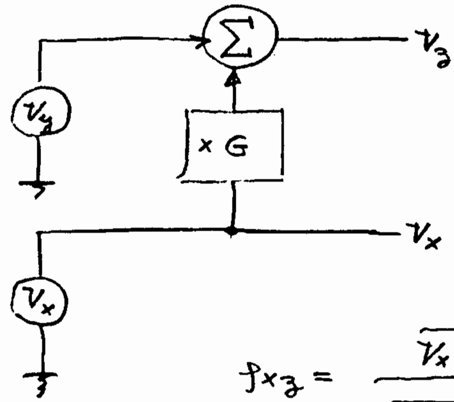
$$V_1 = AV_2 - BI_2 + V_n$$
$$I_1 = CV_2 - DI_2 + I_n$$

SAME PRINCIPLE APPLIED TO (c)
FOUR NUMBERS, CALLED NOISE PARAMETERS DESCRIBE A NOISY NETWORK

ONE SET V_i, V_o, ρ_{io}, ANOTHER SET V_n, I_n, ρ_{ni} (COMPLEX)

25A

EXAMPLE – CORRELATION OF TWO NOISE SOURCES



$$\overline{|V_x|^2} = \overline{|V_y|^2} = V_0^2$$

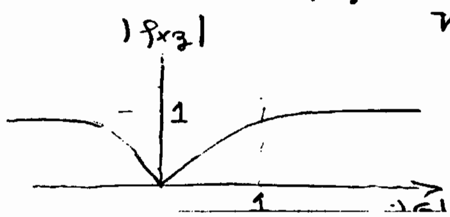
$$\overline{V_x \cdot V_y^*} = 0 \qquad V_z = V_y + G V_x$$

FIND ρ_{xz}

$$\rho_{xz} = \frac{\overline{V_x V_z^*}}{\sqrt{\overline{|V_x|^2}\ \overline{|V_z|^2}}} = \frac{\overline{V_x \cdot V_y^*} + \overline{V_x G^* V_x^*}}{V_0\sqrt{\overline{|V_y + G V_x|^2}}}$$

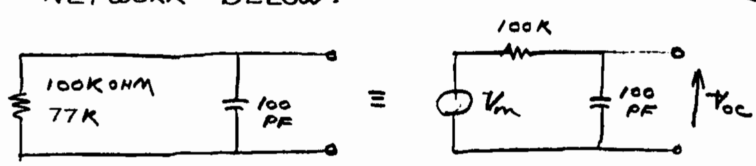
$$\overline{V_x V_y^*} = 0$$

$$\rho_{xz} = \frac{\overline{|V_x|^2}\, G^*}{V_0\sqrt{\overline{|V_y|^2} + |G|^2\,\overline{|V_x|^2}}} = \frac{G^*}{\sqrt{1+|G|^2}}$$



25B

FIND TOTAL WIDEBAND OUTPUT VOLTAGE OF NETWORK BELOW:



$$V_m = \sqrt{4kTR} = \sqrt{4\times1.38\times10^{-23}\times 77\times10^{5}} = 20.6\ mV/\sqrt{Hz}$$

$$V_{oc} = \frac{jX_c}{R+jX_c}V_m = \frac{-j/\omega C}{R-j/\omega C}V_m = \frac{1}{j\omega RC+1}V_m$$

$$|V_{oc}|^2 = \frac{|V_m|^2}{(j\omega RC+1)(-j\omega RC+1)} = \frac{|V_m|^2}{(f/f_0)^2+1} \qquad f_0 = \frac{1}{2\pi RC}$$

$$V_T^2 = \int_0^\infty |V_{oc}|^2 df = |V_m|^2\int_0^\infty \frac{df}{(f/f_0)^2+1} = |V_m|^2 f_0\int_0^\infty \frac{dx}{x^2+1}$$

$$= |V_m|^2 f_0 \tan^{-1}x\Big]_0^\infty = \frac{\pi}{2} f_0\, |V_m|^2$$

$\frac{\pi}{2} f_0$ IS THE NOISE BANDWIDTH OF THE NETWORK

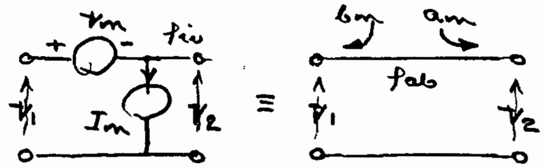
$$V_T^2 = \frac{1.57\times 1}{6.283\times10^5\times10^{-10}}\times(20.6\ mV)^2 = \frac{425\ mV^2}{4\times10^{-5}} = 10.6\ \mu V^2$$

$$V_T = 3.26\ \mu V$$

IF ONE SET OF NOISE PARAMETERS IS KNOWN, ANOTHER SET CAN BE FOUND BY WRITING TWO INDEPENDENT COMPLEX EQUATIONS RELATING TERMINAL VARIABLES AT ONE PORT (i.e. V_1, I_2, a_1, b_2, ETC) WITH THE OTHER PORT TERMINATED IN WHATEVER SIMPLIFIES ANALYSIS (i.e. OPEN, SHORT, OR Z_0).

EXAMPLE — RELATE (c) AND (e)

SINCE NOISELESS NETWORKS ARE THE SAME WE CAN EQUATE ONLY ACTIVE PORTIONS:



EQUATE V_1 WITH Z_0 ON PORT 2 AND OPEN ON PORT 1

$$V_1 = V_m - I_m Z_0 = 2 b_m \sqrt{Z_0}$$

EQUATE V_2 UNDER SAME CONDITIONS

$$V_2 = -I_m Z_0 = (a_m + b_m)\sqrt{Z_0}$$

THUS $b_m = \dfrac{V_m - I_m Z_0}{2\sqrt{Z_0}}$ $\quad a_m = \dfrac{V_m + I_m Z_0}{-2\sqrt{Z_0}}$

f_{ab} CAN ALSO BE FOUND IN TERMS OF f_{in}, V_m, AND I_m

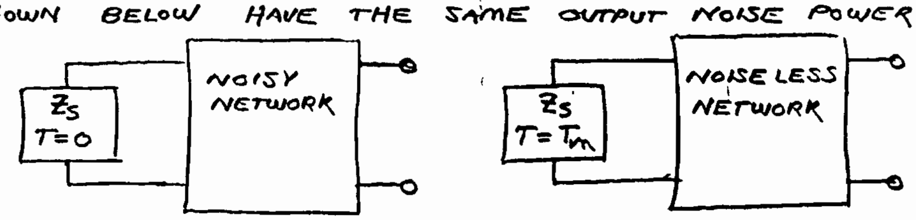
VOLTAGE AND CURRENT NOISE GENERATORS, V_m AND I_m, ARE OFTEN EXPRESSED IN TERMS OF THEIR EQUIVALENT NOISE RESISTANCE, R_m OR CONDUCTANCE, G_m HAVING SAME OUTPUT WHEN AT $T = T_0 \equiv 290K$ $\quad R_m = \dfrac{V_m^2}{4kT_0}$ $\quad G_m = \dfrac{I_m^2}{4kT_0}$

NOISE TEMPERATURE AND OPTIMUM SOURCE IMPEDENCE

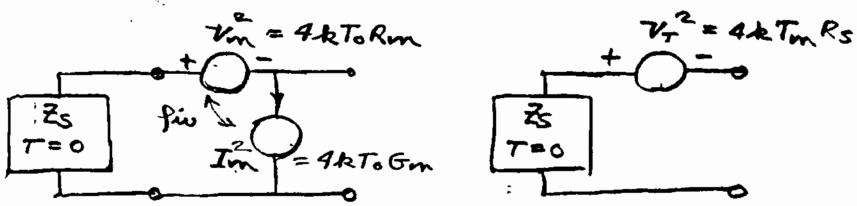
THERE IS A VALUE OF SOURCE (i.e. GENERATOR) IMPEDENCE, Z_S, WHICH MINIMISES THE TOTAL NOISE OUT OF A TWO-PORT NETWORK. THIS OPTIMUM VALUE OF Z_S IS Z_{op}

A CONVENIENT QUANTITY WHICH IS PROPORTIONAL TO THE TOTAL NOISE OUT OF NETWORK IS THE NETWORK NOISE TEMPERATURE $\equiv T_m$ AS SHOWN IN (f).

T_m IS THE TEMPERATURE OF THE SOURCE RESISTANCE WHICH WOULD PRODUCE THERMAL NOISE EQUAL TO THE TOTAL NOISE IN THE NETWORK. THE TWO SOURCE + NETWORK CONFIGURATIONS SHOWN BELOW HAVE THE SAME OUTPUT NOISE POWER:



WE WILL COMPUTE T_m AND Z_{op} IN TERMS OF THE PARAMETERS OF MODEL (c) USING THE EQUIVALENT NOISE RESISTANCE AND CONDUCTANCE, R_m AND G_m, PREVIOUSLY DEFINED



FIND T_m AS A FUNCTION OF R_m, G_m, ρ_{iv}, AND Z_s TO GIVE IDENTICAL SQUARE VOLTAGE FROM ABOVE CIRCUITS

$$V_T^2 = |V_m + I_m Z_s|^2 = |V_m|^2 + |I_m|^2|Z_s|^2 + 2\,\mathrm{Re}\, I_m V_m^* Z_s$$

$$4kT_mR_s = 4kT_0R_m + 4kT_0G_m|Z_s|^2 + 2\times 4kT_0\sqrt{R_mG_m}\,\mathrm{Re}\,\rho_{iv}Z_s$$

$$T_m = \frac{R_m}{R_s}T_0 + G_m|Z_s|^2T_0 + 2\sqrt{R_mG_m}\,T_0\,\mathrm{Re}\,\rho_{iv}Z_s$$

IF $\rho_{iv} = 0$ AS IS THE CASE FOR LOW FREQUENCY (< 10 MHZ) TRANSISTORS

$$T_m = T_0\left(\frac{R_m}{R_s} + G_m|Z_s|^2\right) \quad \text{WITH MIN} \quad T_m = 2T_0\sqrt{R_mG_m} = |V_m||I_m|/2k$$

SOME EXAMPLES

$$Z_s \equiv Z_{op} = \sqrt{R_m/G_m} = \frac{|V_m|}{|I_m|}$$

DEVICE	FREQ	V_m mV/√HZ	I_m fA/√HZ	Z_{op} KOHM	T_{min} °K	
LM741	10 KHZ	20	400	80	145,000	OP AMP
2N4125	100 KHZ	1	1000	1	36	BJT
2N3821	10 KHZ	2.2	3.5	630	0.28	JFET
2N3631	10 KHZ	20	0.2	100,000	0.14	MOSFET

REF
LOW NOISE ELECTRONIC DESIGN
MOTCHENBACHER AND FITCHEN
WILEY, 1973

IN THE CASE $\rho_{iv} \neq 0$ OPTIMUM VALUES ARE:

$$|Z_{op}| = \sqrt{R_m/G_m} \qquad X_{op} = \sqrt{R_m/G_m}\,\mathrm{Im}\,\rho_{iv}$$

$$T_{min} = 2T_0\sqrt{R_mG_m}\left[\mathrm{Re}\,\rho_{iv} + \sqrt{1-(\mathrm{Im}\,\rho_{iv})^2}\right]$$

(SEE VARACTOR APPLICATIONS
PENFIELD AND RAFUSE
p.24)

NOISE TEMPERATURE AS A FUNCTION OF SOURCE IMPEDENCE CAN BE EXPRESSED IN THE FOLLOWING FORMS WHERE $N = R_mG_{op} = G_mR_{op}$, T_{min}, AND Z_{op} ARE FOUR NOISE PARAMETERS DESCRIBING THE NETWORK

$$T_m = T_{min} + NT_0\frac{|Z_s - Z_{op}|^2}{R_sR_{op}}$$

$$T_m = T_{min} + NT_0\frac{|Y_s - Y_{op}|^2}{G_sG_{op}}, \quad Y_s = Z_s^{-1} \quad Y_{op} = Z_{op}^{-1}$$

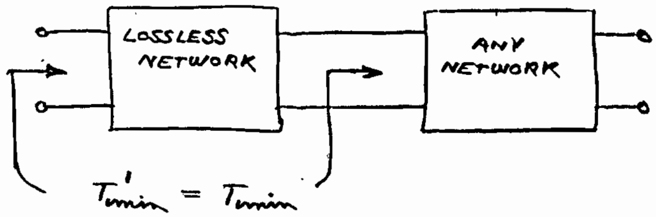
$$T_m = T_{min} + \frac{4NT_0|\Gamma_s - \Gamma_{op}|^2}{(1-|\Gamma_s|^2)(1-|\Gamma_{op}|^2)}, \quad \Gamma_{op} = \frac{Z_{op} - Z_0}{Z_{op} + Z_0}$$

IT CAN BE PROVEN USING THE CONDITION $|\rho| \leq 1$ THAT:

$$T_{min} \leq 4NT_0 \qquad T_0 \equiv 290K \qquad N \geq \frac{T_{min}}{4T_0}$$

N MAY BE CONSIDERED AS THE "CRITICALNESS" FACTOR OF THE NOISE PARAMETERS: NOISE BANDWIDTH $\propto N^{-1}$

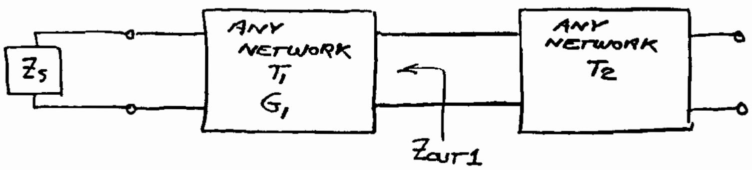
INVARIANCE OF T_{min} AND N THRU LOSSLESS NETWORK



$$T'_{min} = T_{min}$$
$$N' = N$$
$$Z'_{opt} \neq Z_{opt}$$
$$T'_m \neq T_m$$

REF: LANGE, "NOISE CHARACTERIZATION OF LINEAR TWO PORTS IN TERMS OF INVARIANT PARAMETERS" IEEE J. OF SOL. ST. CIR., VOL SC-2, JUNE, 1967 pp. 37-40

NOISE TEMPERATURE CASCADING FORMULA



T_1, T_2 ARE NOISE TEMPERATURES, T_m OF FIRST AND SECOND STAGES

NOISE OF SECOND STAGE CAN BE REFERRED TO INPUT OF FIRST STAGE BY DIVIDING T_2 BY AVAILABLE GAIN, G_1, OF FIRST STAGE. (THE AVAILABLE GAIN IS THE AVAILABLE OUTPUT POWER DIVIDED BY THE AVAILABLE POWER OF A SOURCE OF IMPEDENCE Z_S; IT IS A FUNCTION OF Z_S.)

$$T_{12}\,(\text{OF } Z_S) = T_1\,(\text{OF } Z_S) + \frac{T_2\,[\text{OF } Z_{OUT},\ (\text{OF } Z_S)]}{G_1\,(\text{OF } Z_S)}$$

IN MOST MICROWAVE APPLICATIONS $Z_S = Z_{OUT1} = 50$ AND G_1 = INSERTION GAIN IN 50 OHM SYSTEM

NOISE FIGURE, NF, AND NOISE FACTOR, F

$NF \leq 10 \log F$

NOISE FIGURE, NF, USUALLY EXPRESSED IN DB IS A SIMPLE 1 TO 1 TRANSFORMATION OF T_m

$$NF \equiv 10 \log \left(1 + \frac{T_m}{T_0}\right)$$

$$T_m = T_0\,(10^{NF/10} - 1)$$

NF	T_m
0.5	35K
1.0	75K
2.0	170K
3.0	290K

IT HAS AN ADDITIONAL INTERPRETATION IN TERMS OF THE SIGNAL-TO-NOISE RATIO OUT OF A NETWORK DIVIDED INTO THE SOURCE SIGNAL-TO-NOISE RATIO WITH SOURCE NOISE = 290K

$$NF = 10 \log \frac{(S/N)_{IN}}{(S/N)_{OUT}} \qquad N_{IN} = kT_0$$

(IN MOST CASES $N_{IN} \neq 290K$ SO THIS IS NOT TOO APPLICABLE)

NOISE MEASURE, M

T_m OR NF IS NOT A TOTAL MEASURE OF THE NOISE QUALITY OF A NETWORK SINCE ITS AVAILABLE GAIN, G_a, DETERMINES THE NEXT STAGE CONTRIBUTION. BY ADDING LOSSLESS FEEDBACK TO A NETWORK BOTH T_m AND G_a CAN BE DECREASED BUT A QUANTITY KNOWN AS NOISE MEASURE, M, IS INVARIANT. (FOR EXAMPLE ADDING EXTREME FEEDBACK WHICH CONNECTS OUTPUT DIRECTLY TO INPUT WITH NO OTHER CONNECTION TO THE NETWORK GIVES $T_m = 0$ BUT ALSO $G_a = 1$)

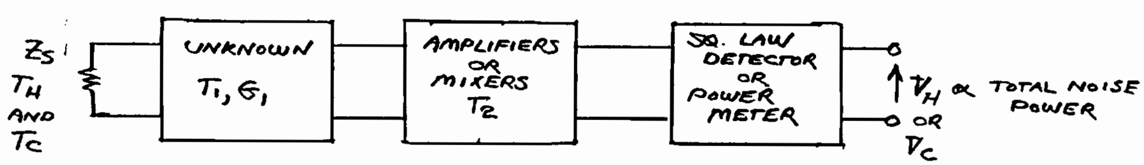
$$M \equiv \frac{T_m/T_0}{1 - 1/G_a} = \frac{F-1}{1-1/G_a}$$

INVARIANCE PROOF REF
HAUS AND ADLER, "OPTIMUM NOISE PERFORMANCE OF LINEAR AMPLIFIERS"
PROC IRE, AUGUST 1958, pp. 1517-1533

NOISE TEMPERATURE, $T_{m\infty}$, OF AN INFINITE CASCADE OF NETWORKS EACH WITH NOISE MEASURE, M, IS MT_0.

$$T_{m\infty} = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \cdots = T_m\left(1 + \frac{1}{G_a} + \frac{1}{G_a^2} + \cdots\right) = \frac{T_m}{1-1/G_a} = MT_0$$

NOISE MEASUREMENT



$$V_H = G\,(T_H + T_{12}) \qquad G = kB \times (\text{TOTAL RECEIVER GAIN}) = \text{VOLTS}/^\circ K$$

$$V_C = G\,(T_C + T_{12}) \qquad T_H \text{ AND } T_C \text{ ARE KNOWN TEMPERATURES}$$

TWO EQS, TWO UNKNOWNS

$$G = \frac{V_H - V_C}{T_H - T_C} \qquad T_{12} = \frac{V_C}{G} - T_C \quad \text{OR} \quad T_{12} = \frac{T_H - T_C}{Y-1} - T_C$$

$$Y \equiv V_H/V_C \equiv Y \text{ FACTOR}$$

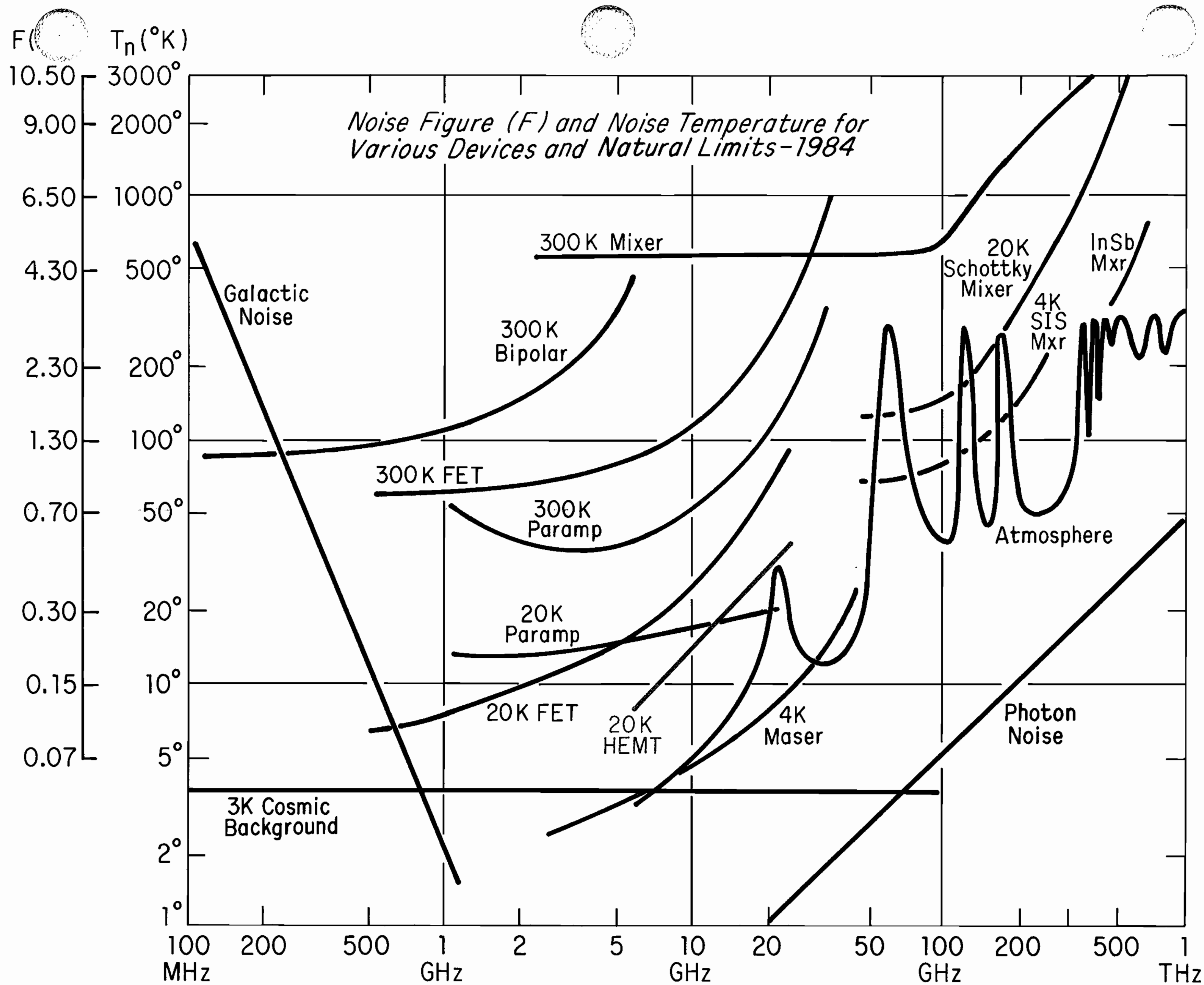
IF $T_2/G_1 \ll T_1$ THEN $T_1 \sim T_{12}$

OR G_1 AND T_2 CAN BE MEASURED BY REMOVING THE FIRST STAGE THEN $T_1 = T_{12} - T_2/G_1$

IF T_1 IS MEASURED AT FOUR VALUES OF Z_S THEN FOUR NOISE PARAMETERS SUCH AS T_{min}, N, AND Z_{opt} OF NETWORK 1 CAN BE DETERMINED.

T_H AND T_C CAN BE PRODUCED BY A SOLID STATE NOISE GENERATOR $\quad T_C = T_{AMBIENT} \quad T_H = T_C + 290 \cdot 10^{ENR/10}$

ENR = EXCESS NOISE RATIO



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TRANSMISSION LINES

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CLASSIFICATION

FIELD DIRECTION	TEM	QUASI - TEM	TE OR TM
TYPICAL SHAPES	COAX STRIPLINE SLAB	MICROSTRIP	RECT CIRC COAXIAL WAVEGUIDES
ϵ, μ IN TRANSVERSE PLANE	UNIFORM	ϵ VARIES	MAY VARY
Z_g, λ_g AS FUNCTIONS OF f	CONSTANT	SMALL VARIATION	LARGE VARIATION
FREQUENCY RANGE FOR PROPAGATION	0 TO ∞	0 TO ∞	$f_{co} < f < \infty$
PHASE VELOCITY, u_p	$c/\sqrt{\epsilon_r}$	$c/\sqrt{\epsilon_r} \le u_p \le c$	$u_p \ge c$

KEY VARIABLES Z_g, γ

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LONGITUDINAL VARIATION OF FIELDS CAN BE PUT IN FORM:

$$V(z) = V_+ e^{-\gamma z} + V_- e^{\gamma z}$$

$$I(z) = \frac{V_+ e^{-\gamma z} - V_- e^{-\gamma z}}{Z_g}$$

SHOULD BE +

WHERE $Z_g \equiv$ CHARACTERISTIC IMPEDENCE

$\gamma \equiv \alpha + j\beta \equiv$ COMPLEX PROPAGATION CONSTANT

$\alpha = \alpha_c + \alpha_D =$ ATTENUATION CONSTANT, CONDUCTOR AND DIELECTRIC

$\beta = 2\pi/\lambda_g$ $\quad \lambda_g \equiv$ GUIDE WAVELENGTH

$u_p = f\lambda_g = 2\pi f/\beta = \omega/\beta =$ PHASE VELOCITY

$\ell =$ PHYSICAL LENGTH $\quad \ell_{EFF} =$ EFFECTIVE (VACUUM) LENGTH

$\tau = \ell/u_p =$ TIME DELAY $= \ell_{EFF}/c \quad \ell_{EFF} = \ell\, c/u_p$

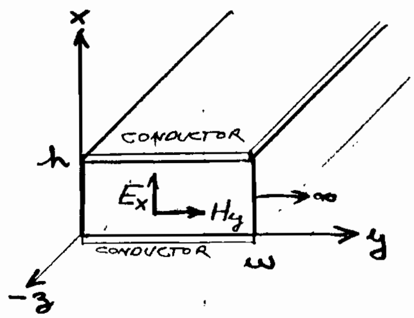
$\theta = \beta\ell = 2\pi\ell/\lambda_g =$ PHASE DELAY (RADIANS)

Z_g, γ ARE OFTEN DIFFICULT TO COMPUTE BUT HAVE BEEN PUBLISHED FOR A WIDE VARIETY OF TRANSMISSION LINES AS FUNCTIONS OF PHYSICAL DIMENSIONS AND ϵ. SEE <u>REF. DATA FOR ENGINEERS</u>, E.C. JORDON 7 TH EDITION, CHAPTERS 29 AND 30

SLAB-LINE – SIMPLE TO ANALYZE AND APPROXIMATES REAL LINES

FIND TEM SOLUTION ($E_z = H_z = 0$) FOR GEOMETRY AND EXCITATION SHOWN ($E_y = 0$)

MAXWELL EQUATION	FOR TEM SLAB-LINE GEOMETRY *
$\nabla \times E = -j\omega\mu H$	$\dfrac{\partial E_x}{\partial z} = -j\omega\mu H_y$
$\nabla \cdot E = \rho/\epsilon$	$\dfrac{\partial E_x}{\partial x} = 0$
$\nabla \times H = (\sigma + j\omega\epsilon)E$	$\dfrac{\partial H_y}{\partial z} = -(\sigma + j\omega\epsilon)E_x$
$\nabla \cdot \mu H = 0$	$\dfrac{\partial H_y}{\partial y} = 0$



*USING $\nabla \times A = \bar{a}_x\left(\dfrac{\partial A_z}{\partial y} - \dfrac{\partial A_y}{\partial z}\right) + \bar{a}_y\left(\dfrac{\partial A_x}{\partial z} - \dfrac{\partial A_z}{\partial x}\right) + \bar{a}_z\left(\dfrac{\partial A_y}{\partial x} - \dfrac{\partial A_x}{\partial y}\right)$

$$\frac{\partial^2 E_x}{\partial z^2} = -j\omega\mu \frac{\partial H_y}{\partial z} = +(\sigma + j\omega\epsilon)(j\omega\mu) E_x$$

SOLVE FOR E_x

THEN $H_y = \dfrac{1}{-j\omega\mu} \cdot \dfrac{\partial E_x}{\partial z}$

SOLUTIONS ARE OF FORM

$$E_x = E_+ e^{-\gamma z} + E_- e^{\gamma z}$$

$$H_y = \frac{E_+ e^{-\gamma z} - E_- e^{\gamma z}}{\eta}$$

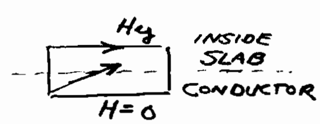
$$V = -\int_0^h E_x \, dx = -h E_x$$

E_+, E_- DETERMINED BY END CONDITIONS

$$\gamma = \sqrt{(\sigma + j\omega\epsilon)(j\omega\mu)}$$
$$= j\omega\sqrt{\epsilon\mu} \quad \text{IF } \sigma = 0$$

$$\eta = \frac{j\omega\mu}{\gamma} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{\mu}{\epsilon}} \quad \text{IF } \sigma = 0$$

THE CURRENT I IN WIDTH w IS FOUND BY



USING $\nabla \times H = J$ IN INTEGRAL FORM

$-H_y \cdot w = I$ TO GIVE:

$$V = V_+ e^{-\gamma z} + V_- e^{\gamma z}$$

$$I = \frac{V_+ e^{-\gamma z} - V_- e^{\gamma z}}{Z_0}$$

$Z_0 = \dfrac{h}{w}\eta$, IF $\sigma = 0$, $\mu = \mu_0$, AND $\epsilon = \epsilon_0$ $\eta = \eta_0 = \sqrt{\dfrac{\mu_0}{\epsilon_0}} = 377$

$Z_0 = 377 \cdot \dfrac{h}{w}$ $\quad$ $Z_0 = 50$ FOR $h/w = 0.133$

SLAB – LINE LOSS – DIELECTRIC, α_D

ATTENUATION DUE TO DIELECTRIC LOSS ($\sigma \neq 0$) IS GIVEN IN THE REAL PART OF THE PROPAGATION CONSTANT

$$\alpha_D = Re\, \gamma = Re \sqrt{(\sigma + j\omega\epsilon)(j\omega\mu)} = Re\, j\omega\sqrt{\epsilon\mu}\sqrt{1+\frac{\sigma}{j\omega\epsilon}} \sim \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$$

$$\alpha_D = \frac{\sigma\eta}{2} \quad \text{WHERE} \quad \eta = \sqrt{\frac{\mu}{\epsilon}} = \frac{\sqrt{\epsilon\mu}}{\epsilon} = \frac{1}{\epsilon v_p} = \frac{2\pi}{\omega\epsilon\lambda_g}$$

$$\alpha_D = \frac{\pi}{\lambda_g}\tan\delta \quad \text{WHERE} \quad \tan\delta = \frac{\sigma}{\omega\epsilon} \equiv \text{DIELECTRIC LOSS TANGENT}$$

(NOTE α IS IN UNITS OF NEPERS/UNIT LENGTH

IN DB $\quad A_{DB} = -10\log P_{OUT}/P_{IN} = -10\log e^{-2\alpha} = 20\alpha\log e = 8.686\,\alpha$)

$$A_{DB} = \frac{27.3\,DB}{\lambda_g}\tan\delta$$

FOR ANY TEM LINE – NOT DEPENDENT UPON GEOMETRY – INCREASES WITH f AT CONSTANT $\tan\delta$

MATERIAL	$\tan\delta$ AT 3 GHz	ϵ
Al_2O_3	.001	8.8 – 10.5
NYLON	.012	2.7 – 3
EPOXY	.027	3.1
QUARTZ	.00006	3.78
TEFLON	.00015	2.1
ETHANOL	.25	6.5

FROM REF. DATA FOR ENG., E.C. JORDON 7NTH ED. p. 4-22

SLAB – LINE LOSS – CONDUCTOR, α_C

CONDUCTOR LOSS MUST BE CALCULATED BY PERTURBATION METHOD (SEE FOUND FOR MIC. ENG., COLLIN, p. 78)

THE MAGNETIC FIELD CALCULATED FOR THE LOSSLESS TEM CASE, H_y, IS USED TO CALCULATE A SURFACE CURRENT, $K_z = H_y$, AND THIS GIVES $E_z = Z_m K_z$ WHERE $Z_m = \dfrac{1+j}{\sigma_m\delta}$ $\quad \sigma_m =$ METAL CONDUCTIVITY

$\delta \equiv$ SKIN DEPTH $= \sqrt{\dfrac{2}{\omega\mu\sigma_m}}$

THE POWER LOST IN AN INCREMENTAL LENGTH, dz, IS $\; dP = \frac{2}{2} Re\, E_z \cdot H_y^* \cdot w \cdot dz = 2\alpha_c P_T\, dz$

WHERE $P_T = \frac{1}{2} Re\, E_x \cdot H_y^* \cdot w \cdot h$

THIS GIVES $\quad \alpha_c = \dfrac{R_m}{\eta h} = \dfrac{R_m}{w Z_g} = \dfrac{R'}{2 Z_g}$

WHERE $R_m = 1/\sigma_m\delta$ AND R' IS THE RESISTANCE PER UNIT LENGTH

$A_{DB} = 8.68\,\alpha_c$

CONDUCTOR	σ_m S/m	f GHz	δ μm	R_m OHMS	A_{DB}/m $Z_g = 50$, $w = 1cm$
COPPER	58×10^6	1	2.1	.008	0.14
"	"	100	0.2	.083	1.39
STAINLESS STEEL	1.1×10^6	1	15.2	.060	1.04
		100	1.52	.60	10.4
GaAs (.01 OHM-CM)	10^4	1	160	.61	10.6
		100	16	6.1	106

$R' = 2\,\dfrac{R_m}{w}$

$\sigma_m = 58 \times 10^6$ S/m FOR Cu

$\delta = \dfrac{2.08\ \text{MICRONS}}{\sqrt{f_{GHz}}} \cdot \sqrt{\dfrac{\sigma_{Cu}}{\sigma_m}}$

$R_m = .0083\sqrt{f_{GHz}} \cdot \sqrt{\sigma_{Cu}/\sigma_m}$

OTHER TEM LINES

ALL TEM LINES HAVE PHASE VELOCITY = GROUP VELOCITY = v_p

$$v_p = \frac{1}{\sqrt{\epsilon \mu}} = \frac{c}{\sqrt{\epsilon_r}} \qquad \lambda_g = \frac{v_p}{f} = \frac{\lambda_0}{\sqrt{\epsilon_r}} \qquad \lambda_0 = c/f$$

$$= 30\,\text{cm}/f_{GHz}$$

$$= 11.80''/f_{GHz}$$

ALL TEM LINES, DIELECTRIC LOSS IS GIVEN BY,

$$\alpha_D = \frac{\pi}{\lambda_g} \tan\delta \qquad \tan\delta = \frac{\sigma}{\omega\epsilon} = \begin{cases} \text{LOSS TANGENT} \\ \text{POWER FACTOR} \\ \text{DISSAPATION FACTOR} \end{cases}$$

$$A_{DB} = 8.68\,\alpha_D = \frac{27.3\,\text{DB}}{\lambda_g} \tan\delta = \frac{27.3\,\text{DB}}{\lambda_0} \cdot \sqrt{\epsilon_r} \cdot \tan\delta$$

NOT A FUNCTION OF LINE DIMENSIONS

ALL TEM LINES, CONDUCTOR LOSS IS GIVEN BY

$$\alpha_c = \frac{R'}{2 Z_g}$$

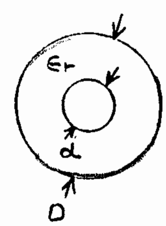
WHERE R' IS THE RESISTANCE PER UNIT LENGTH AND IS A FUNCTION OF LINE SHAPE AND DIMENSIONS

ALL TEM LINES

$$Z_g = \sqrt{\frac{L'}{C'}} \qquad v_p = \frac{1}{\sqrt{L'C'}} = \frac{c}{\sqrt{\epsilon_r}} \qquad L' = \frac{Z_g}{c}\sqrt{\epsilon_r} \qquad C' = \frac{1}{Z_g v_p} = \frac{\sqrt{\epsilon_r}}{Z_g c}$$

WHERE L' AND C' ARE THE STATIC INDUCTANCE AND CAPACITANCE PER UNIT LENGTH

COAX



$$Z_g = \frac{\eta}{2\pi} \ln D/d = \frac{138}{\sqrt{\epsilon_r}} \log D/d$$

D/d	ϵ_r	Z_g
2.303	1	50
3.350	2.1	50
20	1	179.5

LOSS

$$R' = \frac{R_m}{\pi}\left(\frac{1}{d} + \frac{1}{D}\right)$$

COLLIN, p. 81

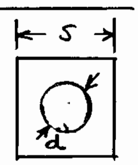
NOTE THAT THIS IS IDENTICAL TO SLAB LINE $R' = \frac{2R_m}{w}$ WITH EQUIVALENT WIDTH $w_{eq} = \frac{2 C_o C_i}{C_o + C_i}$ WHERE C_o, C_i ARE OUTER AND INNER CIRCUMFRENCE.

$\rightarrow C_o$ AS $C_o \sim C_i$

$$A_{DB} = 8.68\,\alpha_c = 4.34\,\frac{R'}{Z_g}$$

$$= 1.38\,\frac{R_m}{Z_g}\left(\frac{1}{d} + \frac{1}{D}\right) \quad \text{DB/(UNIT OF d, D)}$$

SQUARE COAX



FOR $S/d > 1.3$

$$Z_g \sim \frac{138}{\sqrt{\epsilon_r}} \log S/d + 4.54$$

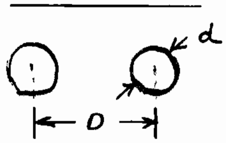
FOR OTHER S/d SEE:

WHEELER, PROC IRE, 38, 1400, 12/50

CRISTAL, PROC IEEE, 52, 10/64

JORDON, REF DATA FOR ENG, p. 29-20

TWIN LEAD

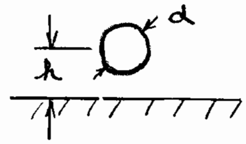


$$Z_g = \frac{\eta}{\pi} \cosh^{-1}(D/d)$$

$$\sim \frac{276}{\sqrt{\epsilon_r}} \log(2D/d) \qquad D/d \gg 1$$

JORDON, REF DATA FOR ENG, 7TH ED. p 29-19

WIRE OVER GROUND PLANE



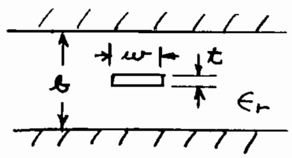
$d \ll h$

$$Z_g = \frac{138}{\sqrt{\epsilon_r}} \log(4h/d)$$

$$L' = \frac{Z_g}{c}\sqrt{\epsilon_r} = \frac{138}{c}\log(4h/d)$$

h/d	Z_g $\epsilon_r = 1$	L' mH/mm
3	149	0.50
10	221	0.74
30	287	0.96

STRIPLINE



SEE DESIGNER'S GUIDE TO STRIPLINE CIRCUITS, BAHL AND GARG, MICROWAVES, 1/78, pp 90-96

APPENDIX C

$\sqrt{\epsilon_r} \times Z_g$ FOR STRIPLINE

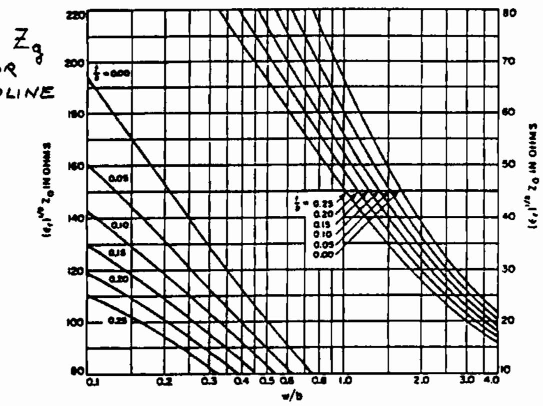


Fig. 30—Plot of strip-transmission-line Z_0 versus w/b for various values of t/b. For lower-left family of curves, refer to left-hand ordinate values; for upper-right curves, use right-hand scale. *Courtesy of Transactions of the IRE Professional Group on Microwave Theory and Techniques.*

MICROSTRIP THEORY (SEE APPENDIX D)

THEORETICAL ANALYSIS OF QUASI-TEM LINES – OUTLINE

WHEN ϵ OR μ VARIES IN TRANSVERSE PLANE AN EXACT, CLOSED FORM SOLUTION OF MAXWELL'S EQUATIONS HAS NOT BEEN FOUND. NOTE DIFFERENT PHASE VELOCITIES IN REGIONS OF DIFFERENT ϵ OR μ AND DIFFICULTY IN SATIFYING BOUNDARY CONDITIONS.

FOR MICROSTRIP PROPAGATION IS PRIMARILY TEM AND THE FOLLOWING APPROXIMATION PROCEDURE HAS BEEN FOUND VALID:

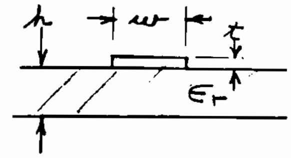
1) AS IN THE TEM CASE, IT IS ASSUMMED THAT

$Z_g = \sqrt{L'/C'}$ AND $v_p = 1/\sqrt{L'C'}$ WHERE L' AND C' ARE THE STATIC INDUCTANCE AND CAPACITANCE PER UNIT LENGTH

2) C' AND ALSO C_0', THE CAPACITANCE PER UNIT LENGTH WITH $\epsilon = \epsilon_0$, ARE CALCULATED EITHER BY CONFORMAL MAPPING OR NUMERICAL METHODS. A "TRICK" IS THEN USED TO FIND L': SINCE L' IS NOT A FUNCTION OF ϵ AND $v_p = c$ FOR $\epsilon = \epsilon_0$ THEN $L'C_0' = 1/c^2$. THEN Z_g AND v_p ARE GIVEN IN TERMS OF $\epsilon_{eff} = C'/C_0'$ BY:

$$Z_g = \sqrt{L'/C'} = \frac{\sqrt{\epsilon_{eff}}}{c\,C'} \qquad v_p = \frac{c}{\sqrt{\epsilon_{eff}}} \qquad \lambda_g = \frac{\lambda_0}{\sqrt{\epsilon_{eff}}}$$

MICROSTRIP Z_g AND v_p



VERY COMPLEX AND ACCURATE VALUES OF Z_g AND $v_p = c/\sqrt{\epsilon_{eff}}$ HAVE BEEN CALCULATED BY SEVERAL AUTHORS.
A RECENT AND COMPLETE SYNOPSIS IS GIVEN BY HAMMERSTAD AND JENSEN. (APPENDIX E). ANOTHER GOOD REFERENCE IS LEV, MICROWAVES AND RF, JAN 1985, pp 111-116. A CAD PROGRAM IS NEEDED FOR ACCURATE VALUES BUT SOME USEFUL GRAPHS FOR THE $t=0$ CASE ARE GIVEN BELOW

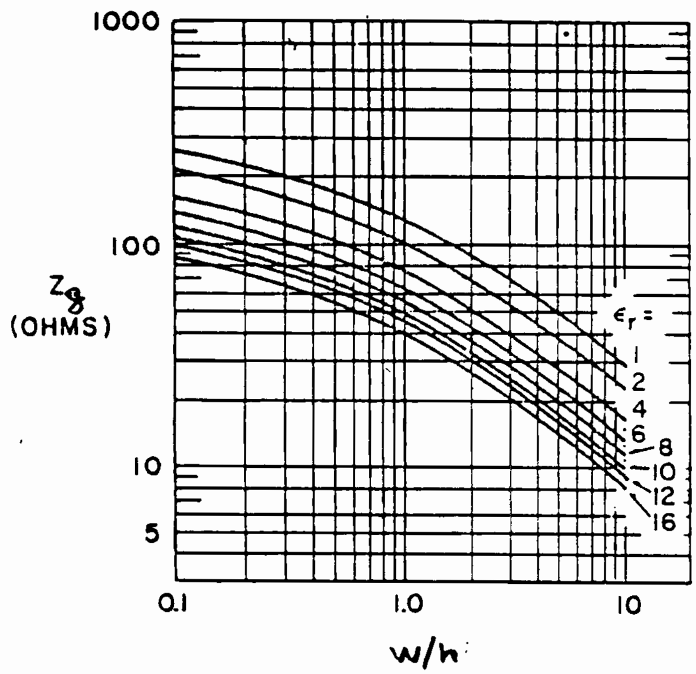


FIG. 4. Impedance of microstrip as function of width of conductor to thickness of substrate. Sobol (2). PROC IEEE, 59, 1971, p. 1200-1211

MICROSTRIP ϵ_{eff}, λ_g, AND Z_g

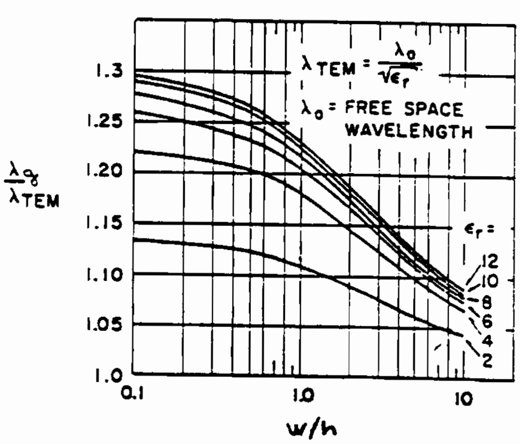
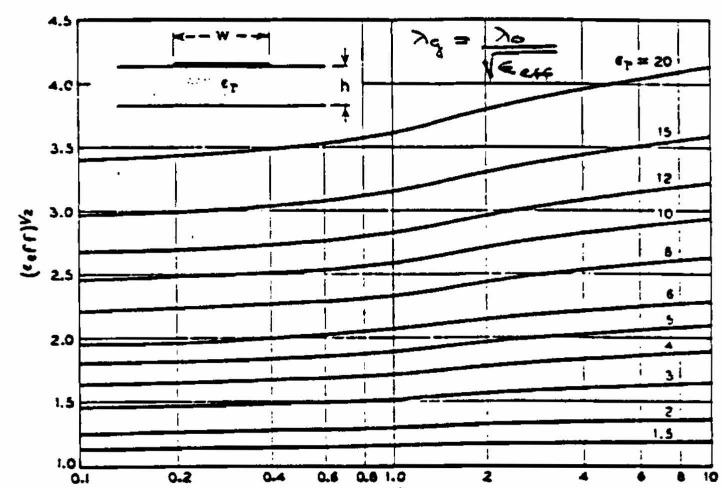


FIG. 5. Normalized guide wavelength of microstrip as function of width of conductor thickness of substrate. Sobol (2). PROC. IEEE, 59, 1971, p. 1200-1211



FROM SCHNEIDER, B.S.T.J, 48 MAY 1969, pp. 1421-44

Fig. 5 — Square root of the effective dielectric constant for the standard microstrip. $(\epsilon_{eff})^{1/2}$ plotted as a function of w/h with ϵ_r as parameter.

THE DIELECTRIC ATTENUATION CONSTANT, α_D, IS GIVEN BY THE SLAB LINE VALUE, $\sigma\eta/2$, MULTIPLIED BY A "FILLING FACTOR" (SCHNEIDER, BSTJ, 48, SEP 1969, P 2325)

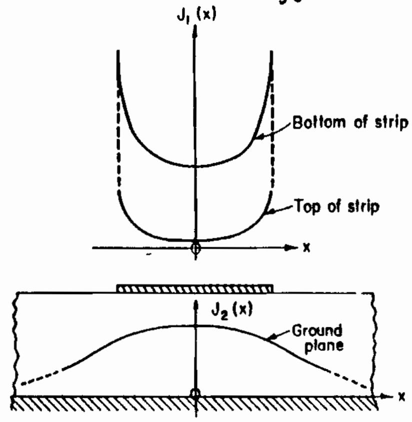
$$\alpha_D = \frac{\sigma\eta}{2} F = \frac{\pi\sqrt{\epsilon_r}}{\lambda_0} \cdot \tan\delta \cdot F \qquad F = \frac{(\epsilon_{eff}-1)}{(\epsilon_r - 1)} \cdot \frac{\epsilon_r}{\epsilon_{eff}}$$

$$\eta = 377/\sqrt{\epsilon_r}$$

THE CONDUCTOR ATTENUATION CONSTANT, α_C, CAN BE CALCULATED BY THE WALL PERTURBATION METHOD (WHEELER, PROC IRE, 30, SEP 1942, AND SCHNEIDER, BSTJ, 48, MAY 1969 P. 1435)

$$A_{DB} = 8.68\,\alpha_C = \frac{R_m\sqrt{\epsilon_{eff}}}{h} \cdot A(w/h)$$

WHERE A IS GIVEN BELOW



Sketch of the current distribution on microstrip conductors.

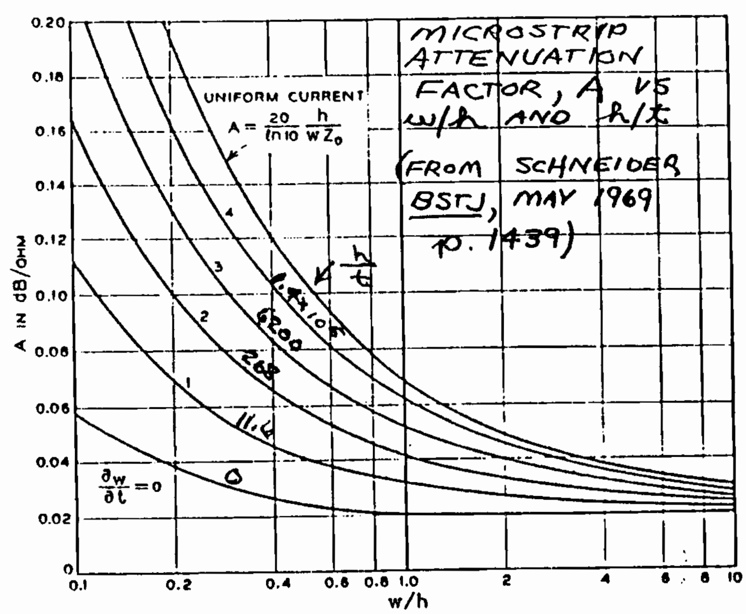
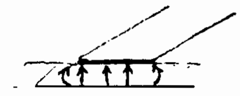
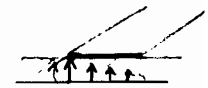


Fig. 8 — Normalized conductor attenuation $A = \alpha_c h/R_s$ in dB per ohm for a standard microstrip with $\epsilon_r = 1$. The partial derivative $\partial w/\partial t$ is a function of the conductor thickness t and given by equations (40) and (41). The conductor attenuation for partial dielectric filling is $\alpha = (\epsilon_{eff})^{1/2}\,\alpha_c$ as given by equation (3).

MICROSTRIP FREQUENCY-SIZE LIMITATIONS

LINE-WIDTH, $w < \lambda_g/4$





AS THE CONDUCTOR WIDTH APPROACHES $\lambda_g/4$ THERE CAN BE UNDESIREABLE TRANSVERSE VARIATION OF FIELD. KEEP $w < \lambda_g/4$ AND EXCITE WIDE LINES AT CENTER
THIS PUT A LOWER LIMIT ON Z_g

SURFACE WAVES, $h < \lambda_0/(4\sqrt{\epsilon_r - 1})$ (HARTWIG, ET AL, 1968 MTT-S DIGEST, PP 110-116)

AS THE SUBSTRATE THICKNESS INCREASES A DIELECTRIC WAVEGUIDE MODE, NOT GUIDED BY THE TOP CONDUCTOR, CAN PROPAGATE AND REMOVE POWER FROM THE CIRCUIT.

WAVEGUIDE MODES, $X < \lambda_0/(2\sqrt{\epsilon_r})$ (GARDIOL, MICROWAVES, MAY 1977 P. 188-191)

IF THE MICROSTRIP IS ENCLOSED IN A METALLIC BOX WAVEGUIDE MODES MAY PROPAGATE IF A TRANSVERSE DIMENSION OF THE BOX, $X \geq \lambda_0/(2\sqrt{\epsilon_r})$. IF THE SUBSTRATE FILLS A SMALL VOLUME OF THE BOX THE LIMIT BECOMES $\lambda_0/2$. THIS LIMIT IS OFTEN EXCEDED WITHOUT DELETERIOUS EFFECTS IF THERE ARE NO DISCONTINUITIES TO LAUNCH THE MODE AND THERE IS LOSSY MATERIAL TO ABSORB THE MODE.

WAVEGUIDE

ANALYSIS IS SIMILAR TO SLAB-LINE TEM CASE ON p 39
EXCEPT EITHER $E_z = 0,\ H_z \neq 0$ (TE SOLUTIONS) OR $H_z = 0,\ E_z \neq 0$ (TM SOLUTIONS) ARE ALLOWED IN THE SOLUTIONS OF MAXWELL'S EQUATIONS. THE RESULTING SOLUTIONS ARE AS IN THE TEM CASE, PRODUCTS OF TRANSVERSE FUNCTIONS $E(x,y)$ AND TRAVELING WAVE FUNCTIONS, $V_+ e^{-\gamma z} + V_- e^{\gamma z}$, EXCEPT THAT MORE COMPLEX x, y VARIATIONS ARE ALLOWED AND γ BECOMES DEPENDENT UPON TRANSVERSE DIMENSIONS, THRU

$$\gamma^2 = -\omega^2 \epsilon \mu + (2\pi/\lambda_c)^2$$

WHERE λ_c IS A CUTOFF WAVELENGTH AND IS A FUNCTION OF MODE NUMBER (n, m) AND DIMENSIONS

USING $\operatorname{Im} \gamma = 2\pi/\lambda_g$

$$\lambda_g = \frac{\lambda_0}{[\epsilon_r - (\lambda_0/\lambda_c)^2]^{1/2}}$$

FOR ANY WAVEGUIDE WHERE $\lambda_0 = c/f$

THE RATIO OF ORTHOGONAL E AND H FIELDS IN THE TRANSVERSE PLANE IS GIVEN BY

$$\frac{E_T}{H_T} = \eta \frac{\lambda_g}{\lambda_0} \quad \text{FOR TE MODES} \qquad \frac{E_T}{H_T} = \eta \frac{\lambda_0}{\lambda_g} \quad \text{FOR TM MODES}$$

WHERE $\eta = \sqrt{\frac{\mu}{\epsilon}}$

RECTANGULAR WAVEGUIDE

FOR A WG OF WIDTH a AND HEIGHT b

$$\lambda_c = \left[\left(\frac{m}{2a}\right)^2 + \left(\frac{n}{2b}\right)^2\right]^{-1/2} \quad \text{FOR } TE_{mn} \text{ OR } TM_{mn} \text{ MODE}$$

(TRANSVERSE ELECTRIC WITH m PEAKS IN WIDTH AND n PEAKS IN HEIGHT)

IF $a > b$ DOMINANT MODE IS TE_{10}

$\lambda_c(TE_{10}) = 2a$

FIELD PATTERNS ARE SIN AND COS DISTRIBUTIONS SHOWN ON FOLLOWING PAGES

Z_g – SOMEWHAT ARBITRARY SINCE V AND I ARE NOT UNIQUE. FOR A PROBLEM NOT ENVOLVING COUPLING TO CIRCUIT ELEMENTS WE MAY ASSUME $Z_g = 1$.
FOR CIRCUIT ELEMENT PROBLEMS A CONVENIENT DEFINITION IS:

$$Z_g = \eta \frac{2b}{a} \frac{\lambda_g}{\lambda_0}$$

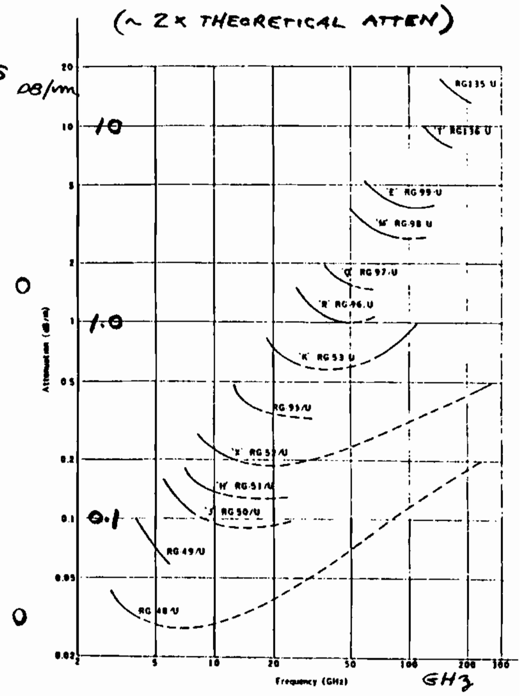
LOSS

$$\alpha_D = \frac{\sigma \eta}{2} \cdot \frac{\lambda_g}{\lambda_0}$$

$$\alpha_C = \frac{R_m}{\eta b} \cdot \frac{\lambda_g}{\lambda_0} \cdot F(m, n, b/a, \lambda_g/\lambda) \qquad F \sim 1$$

SEE COLIN, p. 102 AND FIGURE AT RIGHT

3. EXPERIMENTAL ATTENUATION FOR STANDARD RG RECTANGULAR WAVEGUIDE
(~ 2× THEORETICAL ATTEN)



RECTANGULAR WAVEGUIDE MODE PATTERNS
FROM MARCUVITZ, WAVEGUIDE HANDBOOK

a ≡ WIDTH   b ≡ HEIGHT

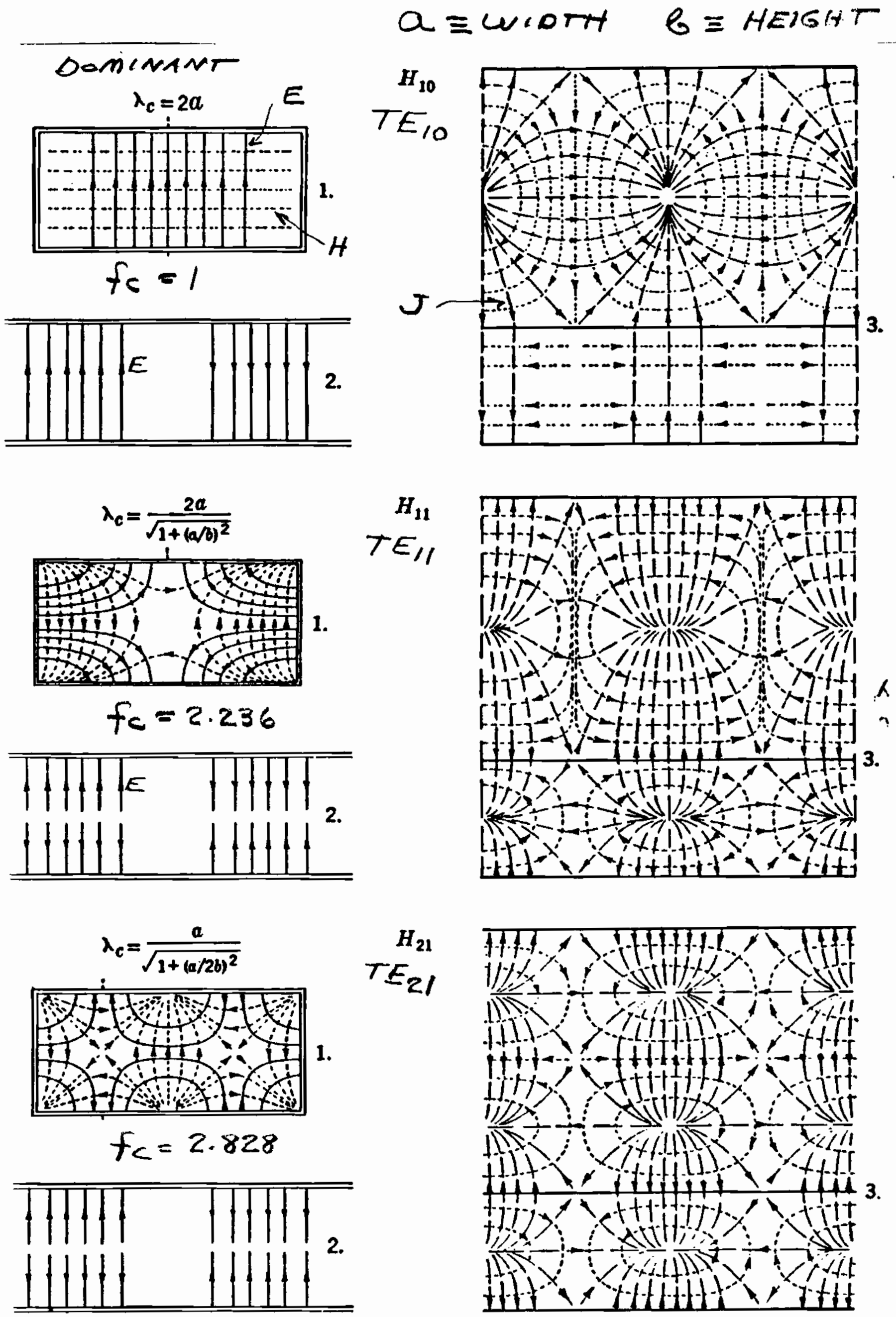


FIG. 2·3.—Field distribution for H-modes in rectangular waveguide.
1. Cross-sectional view
2. Longitudinal view
3. Surface view

$(TE_{01}, TE_{20}$ $\lambda_c = a$
$f_c = 2$

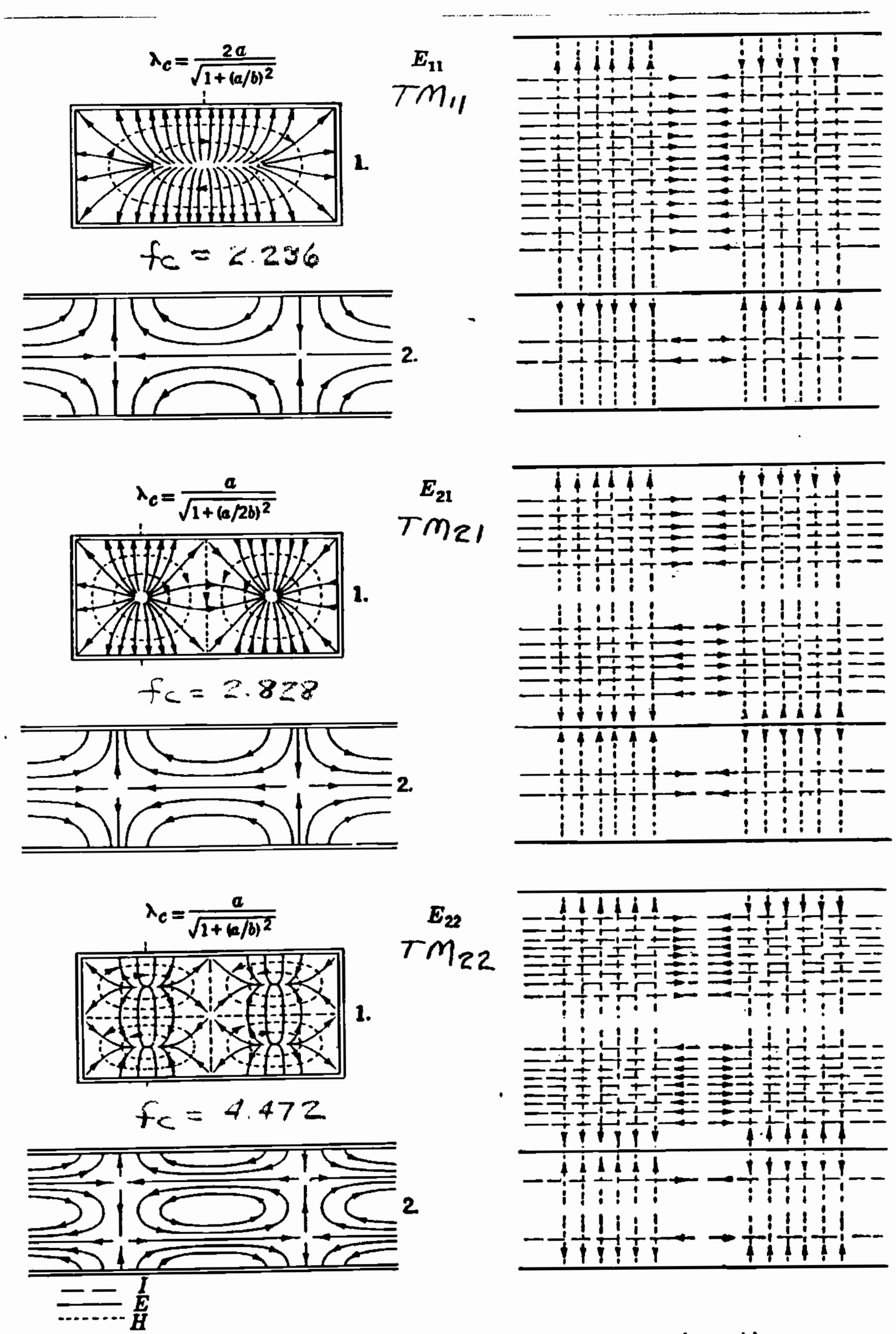


FIG. 2·2.—Field distribution for E-modes in rectangular guides.
1. Cross-sectional view
2. Longitudinal view
3. Surface view

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Reference Table of Rigid Rectangular Waveguide Data and Fittings

EIA WG WR ( ) WR	MDL Band	Recommended Operating Range for TE_{10} Mode: Frequency (kmc/sec)	Recommended Operating Range for TE_{10} Mode: Wavelength (cm)	Cut-off for TE_{10} Mode: Frequency (kmc/sec) f_c	Cut-off for TE_{10} Mode: Wavelength (cm) λ_c	Range in $\frac{2\lambda}{\lambda c}$	Range in $\frac{\lambda_g}{\lambda}$	Theoretical cw power rating lowest to highest frequency (mw)	Theoretical attenuation lowest to highest frequency (db/100 ft.) dB	EIA WG WR ( )	JAN WG RG ( )	Material Alloy	JAN FLANGE Choke UG ( )/U	JAN FLANGE Cover UG ( )/U	DIMENSIONS (Inches) INCHES Inside	Tol. (±)	Outside	Tol. (±)	Wall Thickness (nom.)
2300		0.32-0.49	93.68-61.18	0.256	116.84	1.60-1.05	1.68-1.17	153.0-212.0	.051-.031	2300		Alum.			23.000-11.500	0.020	23.376-11.876	.020	0.188
2100		0.35-0.53	85.65-56.56	0.281	106.68	1.62-1.06	1.68-1.18	120.0-173.0	.054-.034	2100		Alum.			21.000-10.500	0.020	21.376-10.876	.020	0.188
1800		0.41-0.625	73.11-47.96	0.328	91.44	1.60-1.05	1.67-1.18	93.4-131.9	.056-.038	1800	201	Alum.			18.000-9.000	0.020	18.250-9.250	.020	0.125
1500		0.49-0.75	61.18-39.97	0.393	76.20	1.61-1.05	1.62-1.17	67.6-93.3	.069-.050	1500	202	Alum.			15.000-7.500	0.015	15.250-7.750	.015	0.125
1150		0.64-0.96	46.84-31.23	0.513	58.42	1.60-1.07	1.82-1.18	35.0-53.8	.128-.075	1150	203	Alum.			11.500-5.750	0.015	11.750-6.000	.015	0.125
975		0.75-1.12	39.95-26.76	0.605	49.53	1.61-1.08	1.70-1.19	27.0-38.5	.137-.095	975	204	Alum.			9.750-4.875	0.010	10.000-5.125	.010	0.125
770		0.96-1.45	31.23-20.67	0.766	39.12	1.60-1.06	1.66-1.18	17.2-24.1	.201-.136	770	205	Alum.			7.700-3.850	0.010	7.950-4.100	.010	0.125
650	L	1.12-1.70	26.76-17.63	0.908	33.02	1.62-1.07	1.70-1.18	11.9-17.2	.317-.312 .269-.178	650	69 103	Brass Alum.		417A* 418A*	6.500-3.250	0.010	6.660-3.410	.010	0.080
510		1.45-2.20	20.67-13.62	1.157	25.91	1.60-1.05	1.67-1.18	7.5-10.7		510					5.100-2.550	0.010	5.260-2.710	.010	0.080
430	W	1.70-2.60	17.63-11.53	1.372	21.84	1.61-1.06	1.70-1.18	5.2-7.5	.588-.385 .501-.330	430	104 105	Brass Alum.		435A* 437A*	4.300-2.150	0.008	4.460-2.310	.008	0.080
340		2.20-3.30	13.63-9.08	1.736	17.27	1.58-1.05	1.78-1.22	3.1-4.5	.877-.572 .751-.492	340	112 113	Brass Alum.		553* 554*	3.400-1.700	0.005	3.560-1.860	.005	0.080
284	S	2.60-3.95	11.53-7.59	2.078	14.43	1.60-1.05	1.67-1.17	2.2-3.2	1.102-.752 .940-.641	284	48 75	Brass Alum.	54B 585A	53 584	2.840-1.340	0.005	3.000-1.500	.005	0.080
229		3.30-4.90	9.08-6.12	2.577	11.63	1.56-1.05	1.62-1.17	1.6-2.2		229					2.290-1.145	0.005	2.418-1.273	.005	0.064
187	C	3.95-5.85	7.59-5.12	3.152	9.510	1.60-1.08	1.67-1.19	1.4-2.0	2.08-1.44 1.77-1.12	187	49 95	Brass Alum.	148C 406B	149A 407	1.872-0.872	0.005	2.000-1.000	.005	0.064
159		4.90-7.05	6.12-4.25	3.711	8.078	1.51-1.05	1.52-1.19	0.79-1.0		159					1.590-0.795	0.004	1.718-0.923	.004	0.064
137		5.85-8.20	5.12-3.66	4.301	6.970	1.47-1.05	1.48-1.17	0.56-0.71	2.87-2.30 2.45-1.94	137	50 106	Brass Alum.	343B 440B	344 441	1.372-0.622	0.004	1.500-0.750	.004	0.064
112	X_L	7.05-10.0	4.25-2.99	5.259	5.700	1.49-1.05	1.51-1.17	0.35-0.46	4.12-3.21 3.50-2.74	112	51 68	Brass Alum.	52B 137B	51 138	1.122-0.497	0.004	1.250-0.625	.004	0.064
102		7.05-10.0								102	320	Brass	1494	1493	1.020-0.510	0.003	1.148-0.638	.003	0.064
90	X	8.20-12.40	3.66-2.42	6.557	4.572	1.60-1.06	1.68-1.18	0.20-0.29	6.45-4.48 5.49-3.83	90	52 67	Brass Alum.	40B 136B	39 135	0.900-0.400	0.003	1.000-0.500	0.003	0.050
75		10.00-15.00	2.99-2.00	7.868	3.810	1.57-1.05	1.64-1.17	0.17-0.23		75					0.750-0.375	0.003	0.850-0.475	0.003	0.050
62	Ku	12.4-18.0	2.42-1.66	9.486	3.160	1.53-1.05	1.55-1.18	0.12-0.16	9.51-8.31 — — 6.14-5.36	62	91 — 107	Brass Alum. Silver	541A — —	419 — —	0.622-0.311	0.002	0.702-0.391	0.003	0.040
51		15.00-22.00	2.00-1.36	11.574	2.590	1.54-1.05	1.58-1.18	0.080-0.107		51					0.510-0.255	0.0025	0.590-0.335	0.003	0.040
42	K	18.00-26.50	1.66-1.13	14.047	2.134	1.56-1.06	1.60-1.18	0.043-0.058	20.7-14.8 17.6-12.6 13.3-9.5	42	53 121 66	Brass Alum. Silver	596A 598A —	595 597 —	0.420-0.170	0.0020	0.500-0.250	0.003	0.040
34		22.00-33.00	1.36-0.91	17.328	1.730	1.57-1.05	1.62-1.18	0.034-0.048		34		Brass		1530*	0.340-0.170	0.0020	0.420-0.250	0.003	0.040
28	K_A	26.50-40.00	1.13-0.75	21.081	1.422	1.59-1.05	1.65-1.17	0.022-0.031	— — — — 21.9-15.0	28	— — 96	Brass Alum. Silver	600A — —	599 — —	0.280-0.140	0.0015	0.360-0.220	0.002	0.040
22	Q	33.00-50.00	0.91-0.60	26.342	1.138	1.60-1.05	1.67-1.17	0.014-0.020	— — 31.0-20.9	22	— 97	Brass Silver		383 —	0.224-0.112	0.0010	0.304-0.192	0.002	0.040
19		40.00-60.00	0.75-0.50	31.357	0.956	1.57-1.05	1.63-1.16	0.011-0.015		19		Brass		1529*	0.188-0.094	0.0010	0.268-0.174	0.002	0.040
15	V	50.00-75.00	0.60-0.40	39.863	0.752	1.60-1.06	1.67-1.17	0.0063-0.0090	— — 52.9-39.1	15	— 98	Brass Silver		385 —	0.148-0.074	0.0010	0.228-0.154	0.002	0.040
12		60.00-90.00	0.50-0.33	48.350	0.620	1.61-1.06	1.68-1.18	0.0042-0.0060	— — 93.3-52.2	12	— 99	Brass Silver		387 —	0.122-0.061	0.0005	0.202-0.141	0.002	0.040
10		75.00-110.00	0.40-0.27	59.010	0.508	1.57-1.06	1.61-1.18	0.0030-0.0041		10		Brass		1528*	0.100-0.050	0.0005	0.180-0.130	0.002	0.040
8		90.00-140.00	0.333-0.214	73.840	0.406	1.64-1.05	1.75-1.17	0.0018-0.0026	152-99	8	278	Silver Lam.		1527*	0.0800-0.0400	0.0003	0.120-0.080	0.001	0.020
7		110.00-170.00	0.272-0.176	90.840	0.330	1.64-1.06	1.77-1.18	0.0012-0.0017	163-137	7	276	Silver Lam.		1525*	0.0650-0.0325	0.00025	0.105-0.073	0.001	0.020
5		140.00-220.00	0.214-0.136	115.750	0.259	1.65-1.05	1.78-1.17	0.00071-0.00107	308-193	5	275	Silver Lam.		1524*	0.0510-0.0255	0.00025	0.091-0.066	0.001	0.020
4		170.00-260.00	0.176-0.115	137.520	0.218	1.61-1.05	1.69-1.17	0.00052-0.00075	384-254	4	277	Silver Lam.		1526*	0.0430-0.0215	0.00020	0.083-0.062	0.001	0.020
3		220.00-325.00	0.136-0.092	173.280	0.173	1.57-1.06	1.62-1.18	0.00035-0.00047	512-348	3		Silver			0.0340-0.0170	0.00020	0.156 dia.	0.001	—

*Contact Flange

CIRCULAR WAVEGUIDE MODE PATTERNS
FROM MARCUVITZ, WAVEGUIDE HANDBOOK

a = RADIUS

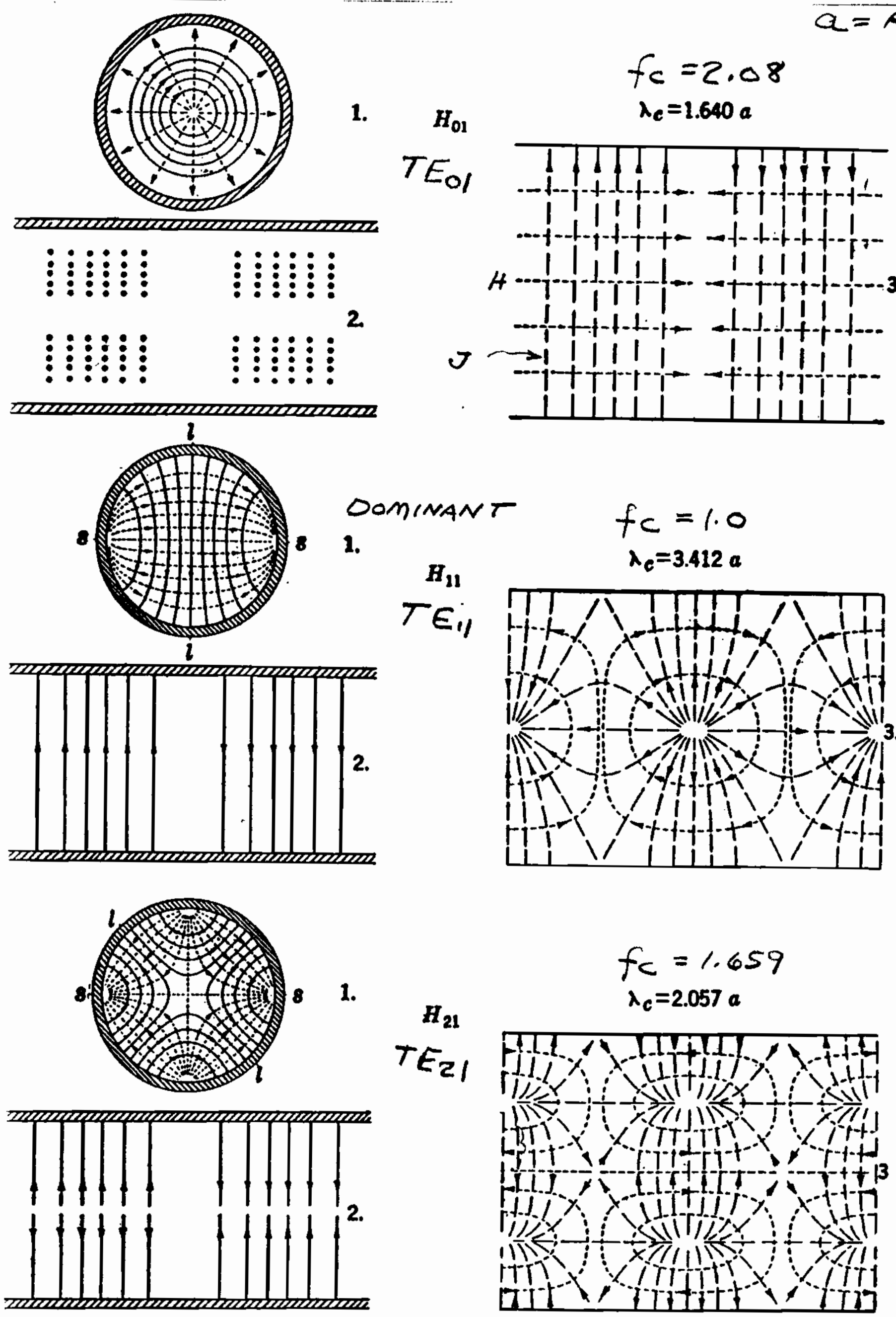


FIG. 2·6.—Field distribution for H-modes in circular waveguide.
1. Cross-sectional view
2. Longitudinal view through plane l-l
3. Surface view from s-s

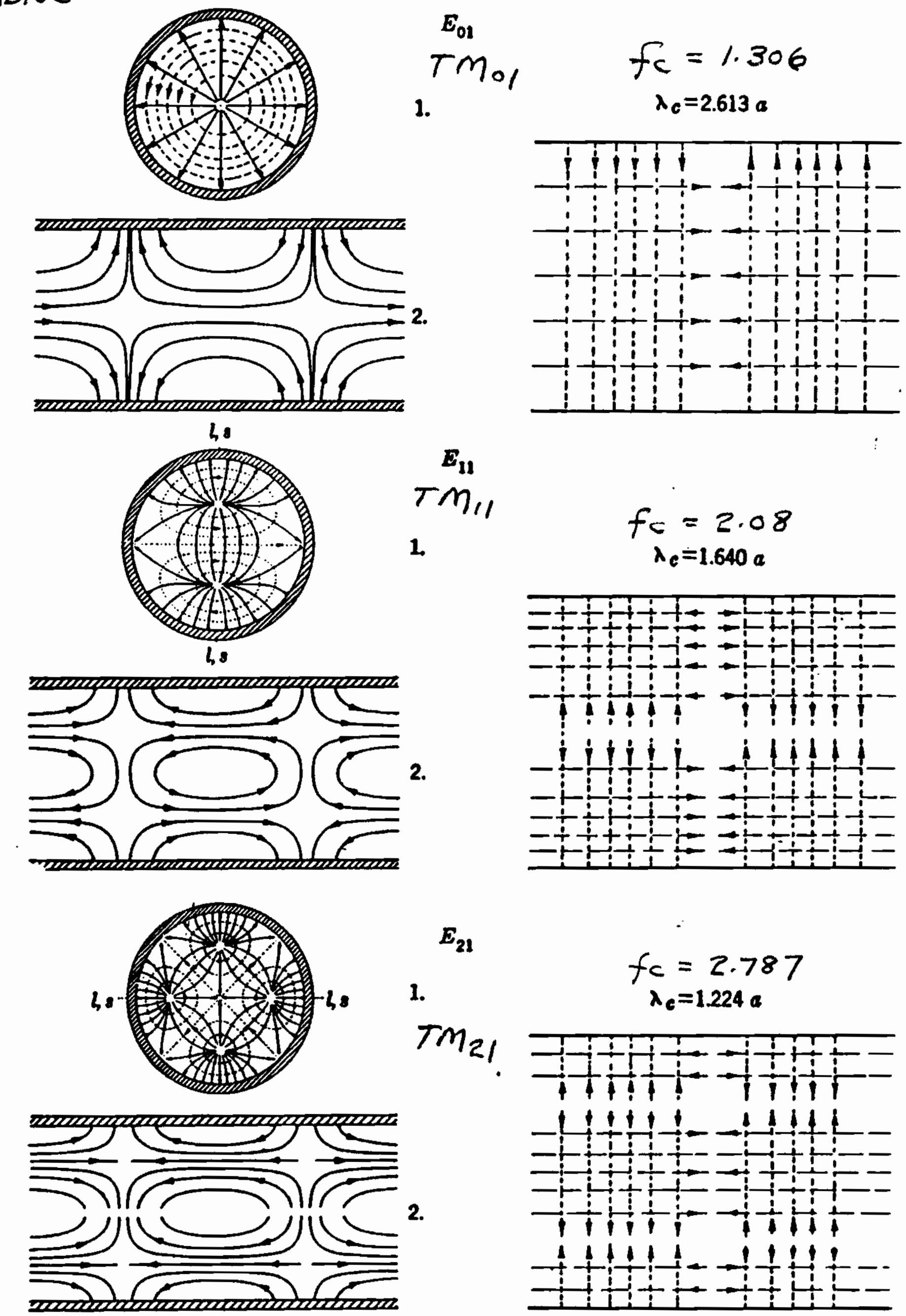


FIG. 2·5.—Field distribution for E-modes in circular waveguide.
1. Cross-sectional view
2. Longitudinal view through plane l-l
3. Surface view from s-s

ATTENUATION OF CUTOFF WAVEGUIDE

WAVE TRAVELING IN z DIRECTION HAS $V(z) = V_+ e^{-\gamma z}$

WITH $\gamma = \sqrt{-\omega^2 \epsilon \mu + (2\pi/\lambda_c)^2}$

$$= 2\pi \sqrt{(1/\lambda_c)^2 - (\epsilon_r/\lambda_0)^2}$$

$$\omega^2 \epsilon \mu = \omega^2 \epsilon_r \epsilon_0 \mu_0 = \frac{(2\pi)^2 f^2 \epsilon_r}{c^2} = \left(\frac{2\pi}{\lambda_0}\right)^2 \epsilon_r$$

$$= \frac{2\pi}{\lambda_c} \sqrt{1 - \epsilon_r (\lambda_c/\lambda_0)^2} = \frac{2\pi}{\lambda_c} \sqrt{1 - \epsilon_r (f/f_c)^2} \qquad f_c = \frac{c}{\lambda_c}$$

WAVEGUIDE IS CUTOFF (γ REAL) IF:

1) WAVELENGTH IN VACUUM, $\lambda_0 > \lambda_c \sqrt{\epsilon_r}$

OR

2) WAVELENGTH IN MEDIA, $\lambda_0/\sqrt{\epsilon_r} > \lambda_c$

LOADING A WAVEGUIDE WITH DIELECTRIC DECREASES THE CUTOFF FREQUENCY. TO PROPAGATE A GIVEN FREQUENCY THE WAVEGUIDE CAN BE SMALLER IF FILLED WITH DIELECTRIC

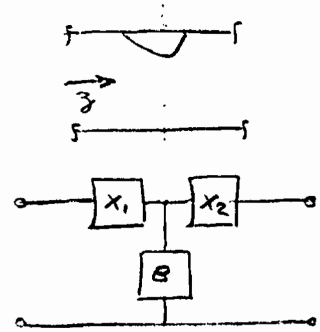
FOR $f << f_c$

$20 \log \gamma = 54.5$ dB PER λ_c

f/f_c	$20 \log \gamma \cdot \lambda_c$ ($\epsilon_r = 1^0$)
0.1	54.2 dB
0.5	47.2 dB
0.9	23.7 dB
0.95	17.0 dB
0.99	7.7 dB
> 1	0

WAVEGUIDE OBSTACLES

USED TO PROVIDE REACTANCES FOR FILTERS AND MATCHING IN WAVEGUIDE. IN MICROSTRIP, STUBS AND SMALL LUMPED ELEMENTS ARE USED FOR THESE PURPOSES.



NUMBER OF ELEMENT VALUES IN EQUIVALENT CIRCUIT:

GENERAL, LOSSLESS CASE – 3

SYMMETRIC, $X_1 = X_2$ – 2

THIN, $X_1 = X_2 = 0$ – 1

ANALYSIS METHOD – MODE MATCHING

THE BOUNDARY CONDITIONS IN THE VICINITY OF THE OBSTACLE ARE MET BY FIELDS WHICH ARE THE SUM OF THE INCIDENT MODE AND OTHER MODES WHICH ARE USUALLY CUTOFF AND TRAVEL IN BOTH DIRECTIONS. THE REFLECTED AND TRANSMITTED INCIDENT MODE AMPLITUDES GIVE S_{11} AND S_{21} AND HENCE THE EQUIVALENT CIRCUIT ELEMENT VALUES.

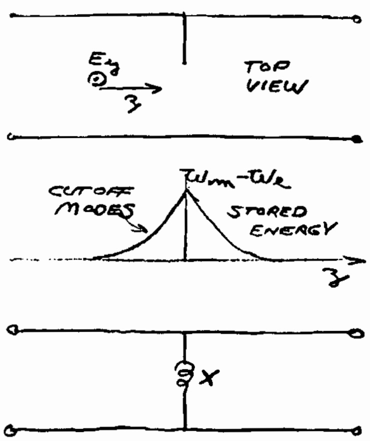
THE INPUT ADMITTANCE OF A STRUCTURE WITH AN OBSTACLE CAN ALSO BE RELATED TO ELECTRIC AND MAGNETIC ENERGY, W_e AND W_m, STORED IN CUTOFF MODES:

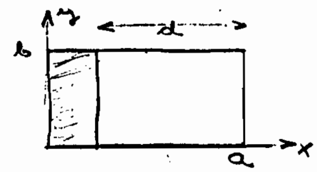
$$\frac{1}{2} Y |V|^2 = P_{01s} + 2j\omega (W_e + W_m)$$

EXAMPLE: THIN INDUCTIVE IRIS WITH INCIDENT TE_{10} MODE

THE CONDITION OF ZERO E_y ON THE CONDUCTING VANE IS MET WITH A SUM OF TE_{m0} MODES.

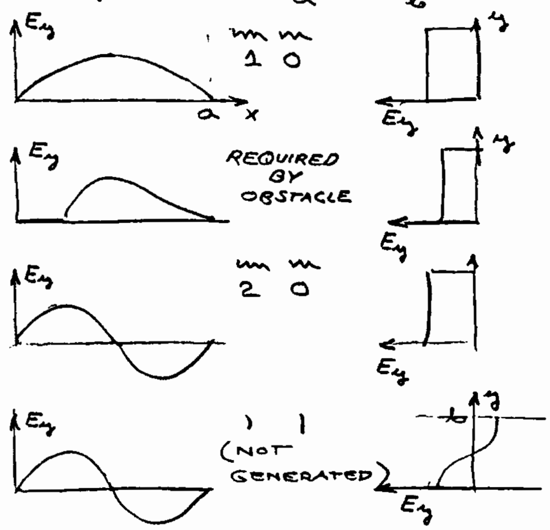
THESE STORE AN EXCESS OF MAGNETIC ENERGY AND THE EQUIVALENT REACTANCE, X, IS INDUCTIVE.





TE_{mn}

$E_y \propto m \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b}$



SPATIAL FOURIER SERIES

OBSTACLE EQUIVALENT CIRCUITS (FROM MARCUVITZ, WG HANDBOOK, M.I.T. RAD LAB VOL 10) (62)

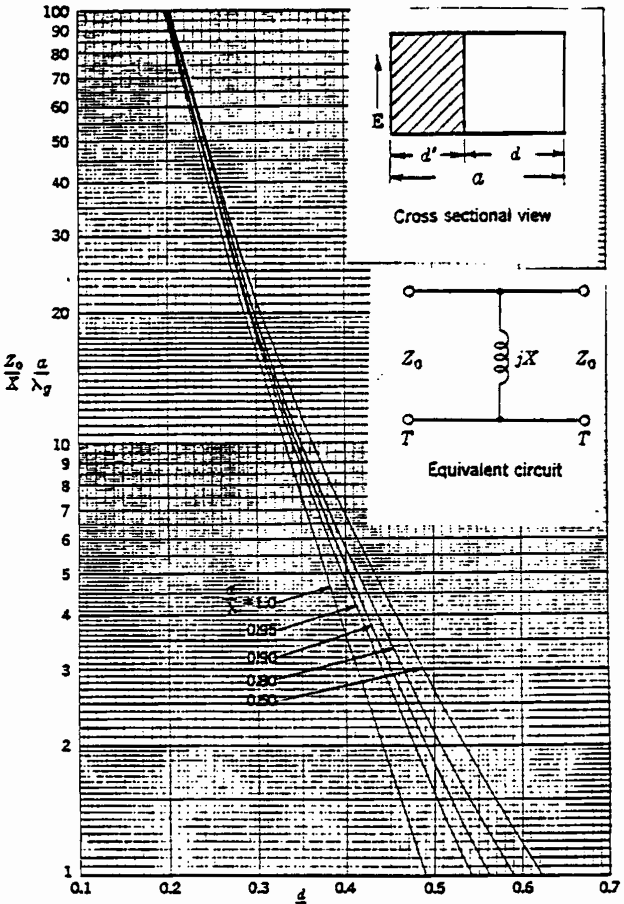


FIG. 5·2-5.—Susceptance of asymmetrical inductive window in rectangular guide.

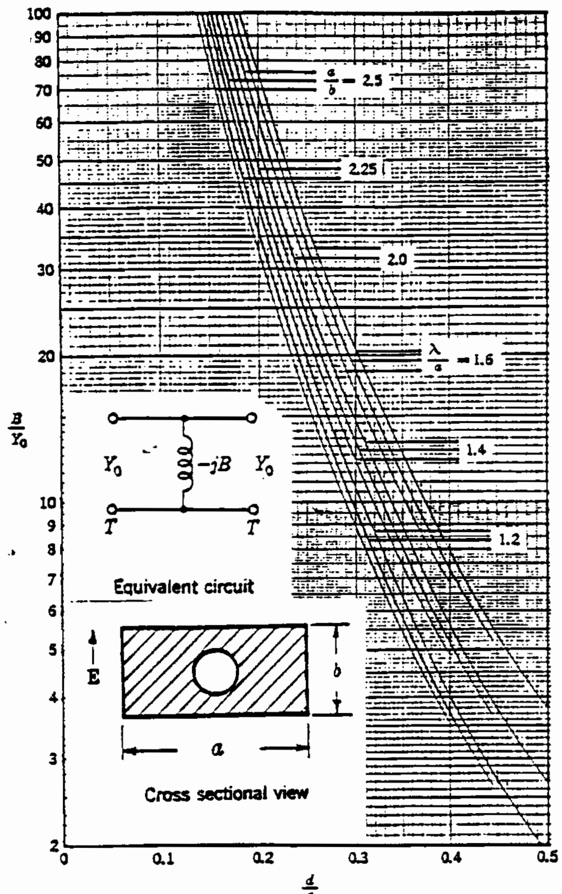


FIG. 5·4-2.—Relative susceptance of centered circular aperture.

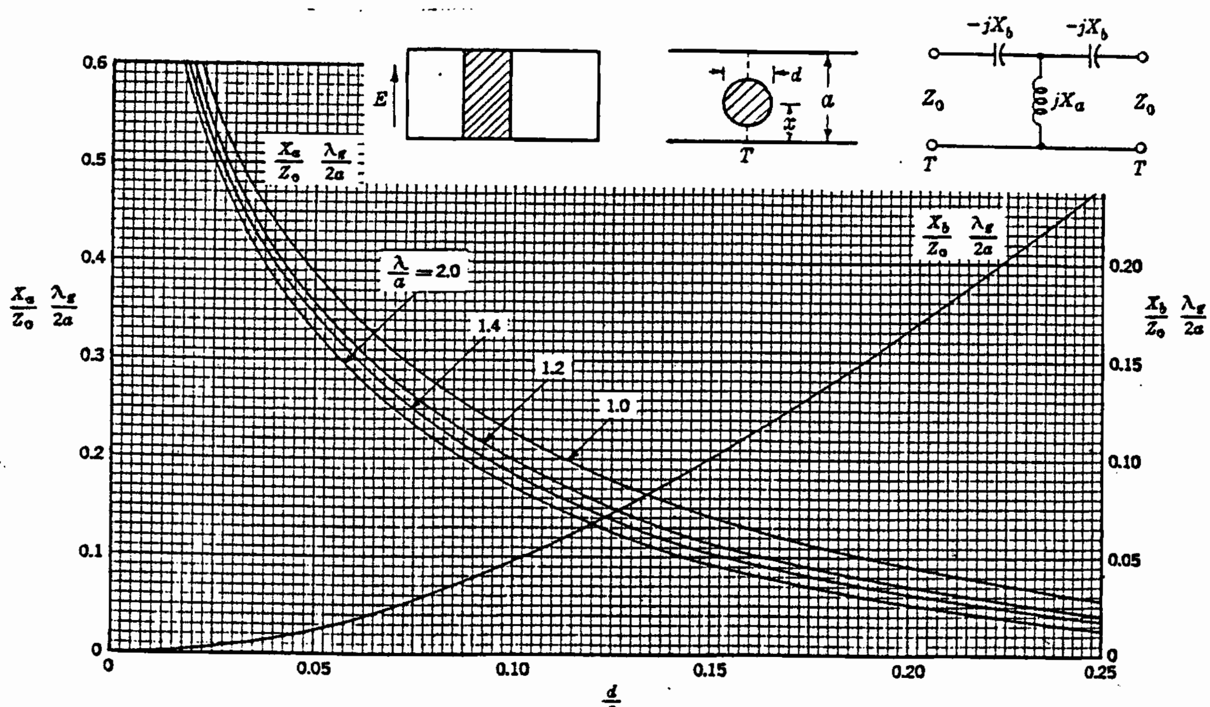


FIG. 5·11-5.—Circuit parameters of centered inductive post in rectangular guide.

THICK POST IN RECTANGULAR WAVEGUIDE
(FROM MARCUVITZ, p. 271)

5-14. Post of Variable Height in Rectangular Guide.—A centered metallic cylindrical post of variable height with axis parallel to the dominant-mode electric field (H_{10}-mode in rectangular guide).

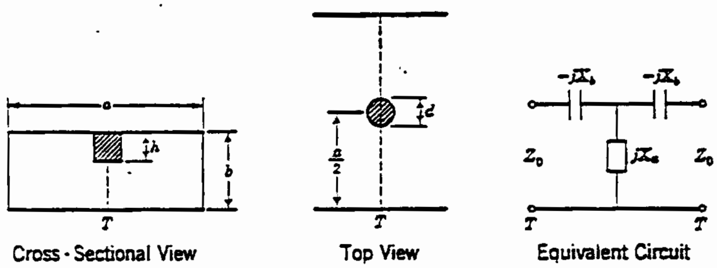
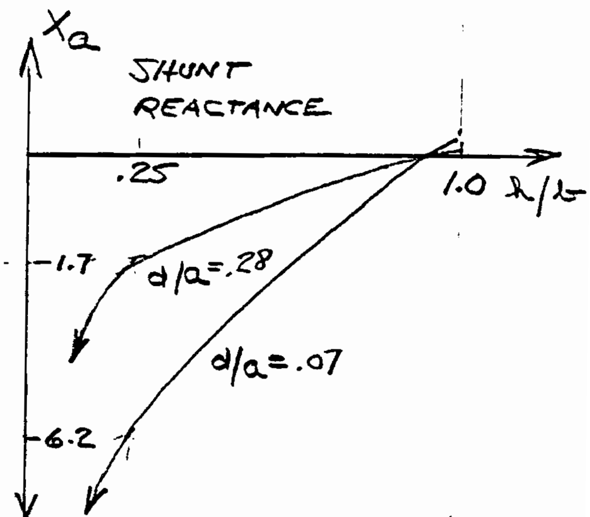
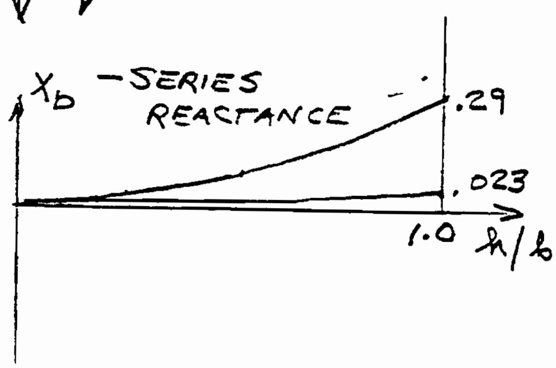


FIG. 5-14-1.

Equivalent-circuit Parameters. Experimental.—The equivalent-circuit parameters at the terminal plane T have been measured in rectangular guide of dimensions $a = 0.90$ in. and $b = 0.40$ in. The measured data are tabulated below as a function of h/b for a number of post diameters d and wavelengths λ. For posts with a flat base:





d/a = .07

$d = \frac{1}{16}$ in., $\lambda = 3.4$ cm, $\lambda_g = 2.000$ in.

h/b (in.)	0.249	0.497	0.746	0.871	0.921	0.934	0.993	1.000
X_b/Z_0	0.005	0.010	0.014	0.017	0.018	0.018	0.020	0.020
X_a/Z_0	−6.481	−1.015	−0.894	−0.035	+0.016	0.031	0.151	0.241

$d = \frac{1}{16}$ in., $\lambda = 3.2$ cm, $\lambda_g = 1.763$ in.

h/b (in.)	0.254	0.505	0.756	0.829	0.943	0.961	1.000
X_b/Z_0	0.006	0.011	0.017	0.019	0.021	0.022	0.023
X_a/Z_0	−6.204	−0.906	−0.122	−0.028	+0.083	0.112	0.277

$d = \frac{1}{16}$ in., $\lambda = 3.0$ cm, $\lambda_g = 1.561$ in.

h/b (in.)	0.246	0.504	0.629	0.755	0.784	0.845	0.898	1.000
X_b/Z_0	0.005	0.013	0.016	0.019	0.019	0.021	0.022	0.025
X_a/Z_0	−6.384	−0.763	−0.277	−0.053	−0.017	+0.047	0.088	0.341

d/a = .14

$d = \frac{1}{8}$ in., $\lambda = 3.4$ cm, $\lambda_g = 2.001$ in.

h/b (in.)	0.258	0.507	0.758	0.882	0.970	1.000
X_b/Z_0	0.016	0.035	0.054	0.065	0.073	0.076
X_a/Z_0	−3.179	−0.606	−0.147	−0.052	+0.028	0.107

$d = \frac{1}{8}$ in., $\lambda = 3.2$ cm, $\lambda_g = 1.764$ in.

h/b (in.)	0.251	0.501	0.759	0.834	0.882	0.965	1.000
X_b/Z_0	0.017	0.038	0.061	0.068	0.073	0.081	0.085
X_a/Z_0	−3.37	−0.591	−0.129	−0.058	−0.020	+0.040	0.126

$d = \frac{1}{8}$ in., $\lambda = 3.0$ cm, $\lambda_g = 1.561$ in.

h/b (in.)	0.240	0.488	0.745	0.818	0.923	1.000
X_b/Z_0	0.019	0.044	0.069	0.077	0.086	0.098
X_a/Z_0	−3.333	−0.596	−0.109	−0.050	+0.027	0.147

d/a = .28

$d = \frac{1}{4}$ in., $\lambda = 3.4$ cm, $\lambda_g = 2.000$ in.

h/b (in.)	0.252	0.499	0.760	0.925	1.000
X_b/Z_0	0.047	0.101	0.174	0.227	0.256
X_a/Z_0	−1.775	−0.468	−0.166	−0.053	+0.026

$d = \frac{1}{4}$ in., $\lambda = 3.2$ cm, $\lambda_g = 1.761$ in.

h/b (in.)	0.262	0.505	0.755	0.880	0.924	1.000
X_b/Z_0	0.052	0.111	0.191	0.240	0.267	0.291
X_a/Z_0	−1.717	−0.477	−0.182	−0.088	−0.038	+0.033

$d = \frac{1}{4}$ in., $\lambda = 3.0$ cm, $\lambda_g = 1.561$ in.

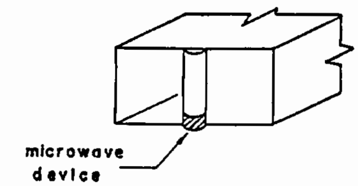
h/b (in.)	0.250	0.502	0.750	0.880	0.940	1.000
X_b/Z_0	0.056	0.121	0.211	0.270	0.300	0.335
X_a/Z_0	−1.859	−0.494	−0.179	−0.085	−0.040	+0.023

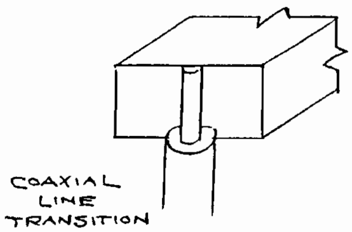
WAVEGUIDE TO MICROWAVE DEVICE OR COAX

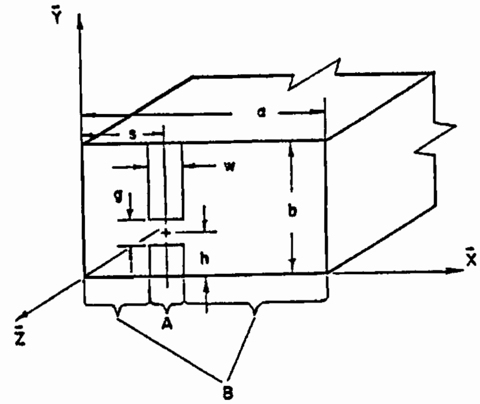
CLASSIC PAPER IS EISENHART AND KAHN (MTT-19, AUG 1971 PP 706-719)

WAS EXTENDED TO COAXIAL TRANSITIONS BY EISENHART, ET AL (MTT-26, MAR 1978)

MULTIPLE POSTS AND MULTIPLE GAPS ARE ANALYZED BY JOSHI AND CORNICK (MTT-25, MAR 1977)







(b)

- a width of waveguide
- b height of waveguide
- h gap position (center from bottom)
- s post position (center from side)
- w post width (flat strip)
- g gap size
- $s' = s/a$ normalized post position
- $w' = w/a$ normalized post width
- $h' = h/b$ normalized gap position
- $g' = g/b$ normalized gap size.

EQUIVALENT CIRCUIT BETWEEN GAP AND WAVEGUIDE IS SHOWN AT RIGHT. THE TRANSFORMER TURNS-RATIO'S ARE GIVEN AS GAP COUPLING COEFFICIENTS

$$K_{gn} = \underbrace{\cos\frac{n\pi h}{b}}_{\text{GAP POSITION} = h} \cdot \underbrace{\frac{\sin\frac{n\pi g}{2b}}{\frac{n\pi g}{2b}}}_{\text{GAP SIZE} = g}$$

DEPENDS ON:

AND POST COUPLING COEFFICIENTS

$$K_{pm} = \underbrace{\sin\frac{m\pi s}{a}}_{\text{POST POSITION} = s} \cdot \underbrace{\frac{\sin\frac{m\pi w}{2a}}{\frac{m\pi w}{2a}}}_{\text{POST WIDTH} = w}$$

DEPENDS ON

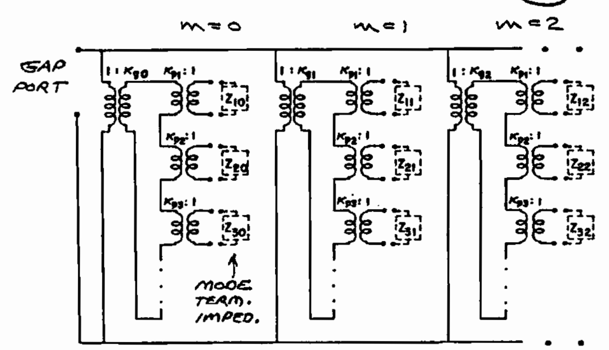
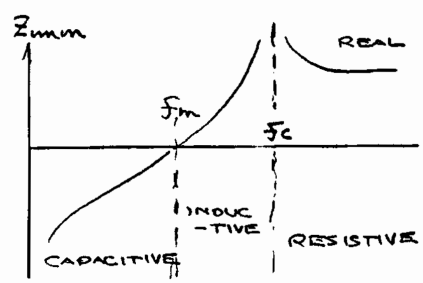


Fig. 5. Equivalent circuit of post mount.



THE MODE PAIR IMPEDANCES, Z_{mn}, ARE THE IMPEDANCES FACING BOTH DIRECTIONS IN THE WAVEGUIDE FOR TE_{mn} AND TM_{mn} MODES.

FOR CUTOFF MODES $Z_{mn} = Z_{g\,mn}/2$ AND IS REACTIVE

FOR PROPAGATING MODES IT IS THE TERMINATING IMPEDANCE TRANSFORMED BY LINE LENGTH

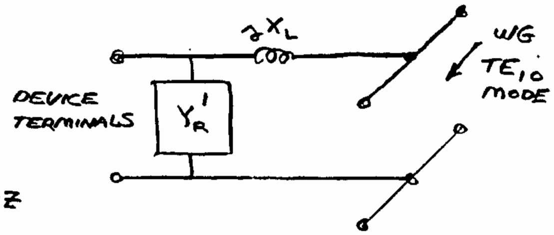
$$Z_{mn} = j m \frac{b}{a} \frac{1}{2-\delta_0} \cdot \frac{1-(f_m/f)^2}{\sqrt{(f_c/f)^2-1}}$$

$$f_m = \frac{mc}{2b} \qquad f_c^2 = \left[\left(\frac{m}{2a}\right)^2 + \left(\frac{n}{2b}\right)^2\right] c^2 \qquad \delta_0 = \begin{matrix} 1 & m=0 \\ 0 & m\neq 0 \end{matrix}$$

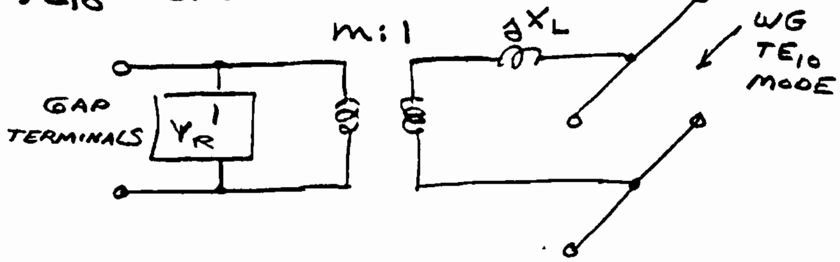
TE_{10} MODE EQUIVALENT CIRCUIT

NOTE THAT FOR A SHORT-CIRCUIT AT THE DEVICE TERMINALS, X_L IS JUST THE POST OBSTACLE INDUCTANCE AS GIVEN BY MARCUVITZ

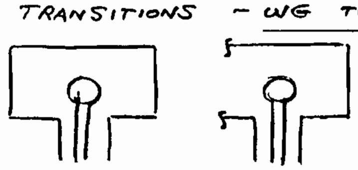
PLOTS GIVING Y_R' AND X_L VS GAP DIMENSIONS AND FREQUENCY ARE GIVEN BY EISENHART



TE_{10} CIRCUIT IS:

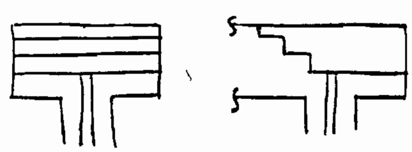


TRANSITIONS – WG TO COAX



VERY COMMON

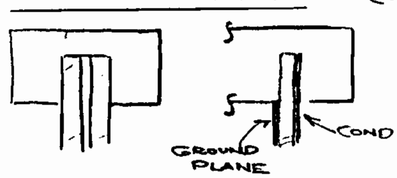
$|\Gamma| < 0.15$ OVER FULL WG BAND



REDUCED HEIGHT WG

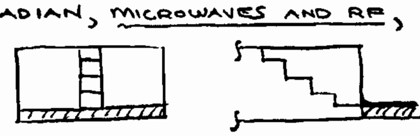
$|\Gamma| \leq .05$ FULL WG BAND

WG TO MICROSTRIP (SEE IZADIAN, MICROWAVES AND RF, MAY 1987, p. 213)

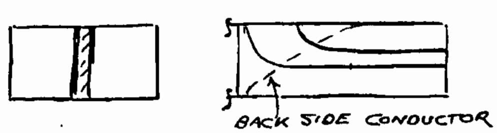


PROBE TRANSITION

$|\Gamma| < 0.1$ FULL WG

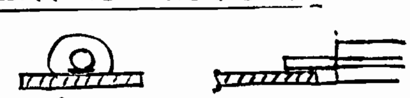


RIDGE WAVEGUIDE
– IN LINE MICROSTRIP
(SEE MAZILU, MIC. JOURNAL, JULY 1987, p. 133)
$|\Gamma| < 0.1$ FULL WG



WG – FINLINE – MICROSTRIP
(VAN HEUVEN, IEEE MTT, MARCH 1976)

COAX TO MICROSTRIP



BUTT JOINT IN LINE

$|\Gamma| < 0.1$ IF CENTER CONDUCTOR ~ MICROSTRIP WIDTH

TRANSMISSION LINES IN THE TIME DOMAIN

- WHY IMPORTANT?
  1) FOR ANALYSIS OF PULSE SIGNALS AS IN HIGH SPEED COMPUTERS
  2) TIME-DOMAIN REFLECTOMETERS
  3) TIME-GATING AS APPLIED IN NETWORK ANALYZERS (HP8510)
- TWO APPROACHES
  1) DIRECT IMPULSE OR STEP TIME-DOMAIN ANALYSIS
  2) FOURIER TRANSFORM OF SINUSOIDAL STEADY STATE ANALYSIS

DIRECT - TIME DOMAIN ANALYSIS

MAXWELL'S EQUATIONS ARE WRITTEN IN TIME DERIVATIVE FORM [I.E. $\nabla \times \mathcal{E}(t) = -\frac{\partial}{\partial t} \mu \mathcal{H}(t)$]

THE WAVE EQUATION ON A TRANSMISSION LINE BECOMES

$$\frac{\partial^2 v}{\partial z^2} = \epsilon\mu \frac{\partial^2 v}{\partial t^2}$$

WHERE $v(t,z)$ IS THE VOLTAGE ON THE LINE

WITH SOLUTIONS:

$$v(t,z) = v_+(t-\tau) + v_-(t+\tau)$$

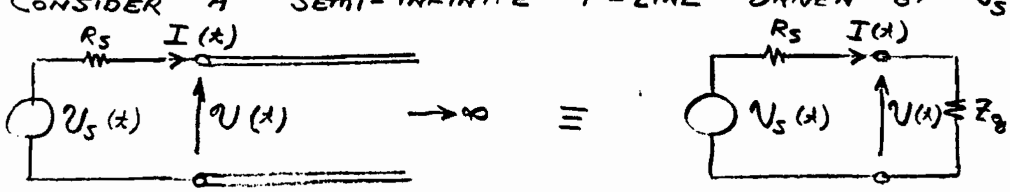
$$Z_g i(t,z) = v_+(t-\tau) - v_-(t+\tau).$$

$\tau = z/v$

$v = \frac{1}{\sqrt{\epsilon\mu}}$

$v_+(t)$ AND $v_-(t)$ ARE ARBITRARY FUNCTIONS

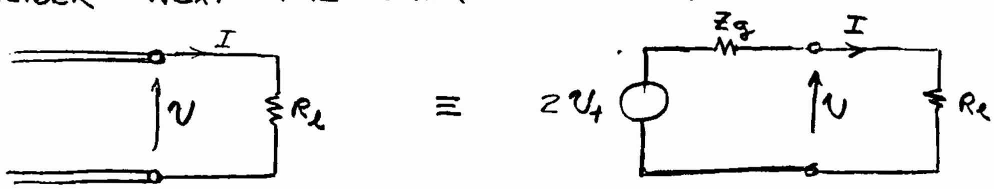
CONSIDER A SEMI-INFINITE T-LINE DRIVEN BY $v_s(t)$ 70



$v_-(t) = 0$ FOR $t <$ ROUND-TRIP DELAY TO A REFLECTIONS

$$v_+(t) = \frac{Z_g}{Z_g + R_s} v_s(t)$$

CONSIDER NEXT THE OTHER END OF THE LINE



THEVENIN EQUIVALENT IN TIME DOMAIN

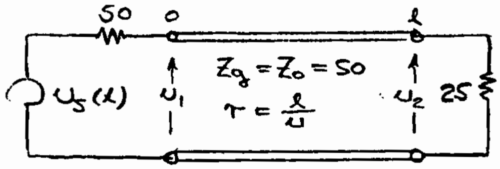
$$v_+ + v_- = v$$

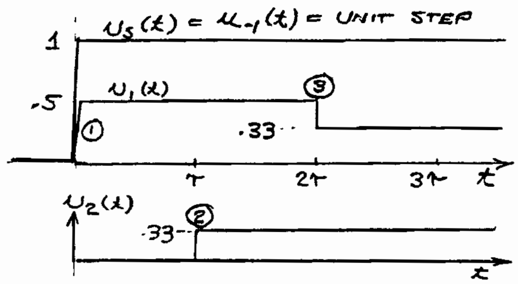
$$v_+ - v_- = \frac{v Z_g}{R_\ell}$$

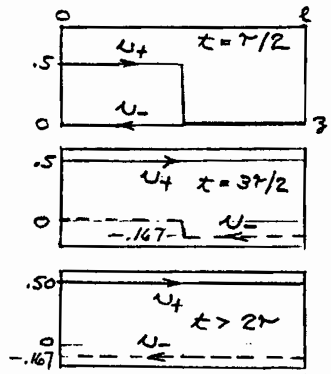
THEN $v = 2v_+ R_\ell/(R_\ell + Z_g)$

$$v_- = v_+ \cdot \frac{R_\ell - Z_g}{R_\ell + Z_g}$$

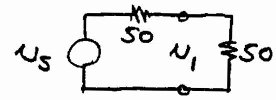
EXAMPLE – STEP FUNCTION RESPONSE OF TRANSMISSION-LINE TERMINATED WITH A RESISTOR





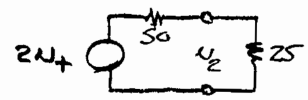


① USE SOURCE-LINE EQUIVALENT CIRCUIT TO FIND $v_1(t) = v_+(t,0) + v_-(t,0)$. NOTE $v_-(t,0) = 0$ FOR $t \le 2\tau$ SINCE LINE IS INITIALLY DEAD AND THAT $v_1(t)$ WILL BE AFFECTED BY v_- UNTIL $t \ge 2\tau$.



$v_s = u_{-1}(t)$
$v_1 = \frac{1}{2} u_{-1}(t)$
$0 \le t \le 2\tau$

② USE LINE-LOAD EQUIVALENT CIRCUIT TO FIND $v_2(t)$ AND $v_-(t,\ell) = v_2(t) - v_+(t,\ell)$

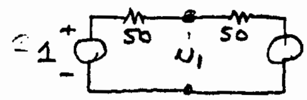


$v_2 = \frac{1}{3} \cdot 2v_+$
$= .33$
$v_- = -.167$
$0 \le t \le \tau$

ALTERNATIVE IF LOAD IS RESISTIVE:

$v_- = \Gamma_\ell v_+$ WHERE $\Gamma_\ell = \frac{R_\ell - Z_g}{R_\ell + Z_g} = -.33$

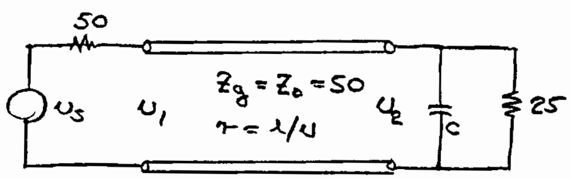
③ USE LINE-LOAD CIRCUIT AGAIN AT $z = 0$ TO FIND $v_1(t)$ AND $v_+(t,0)$ FOR $t \ge 2\tau$

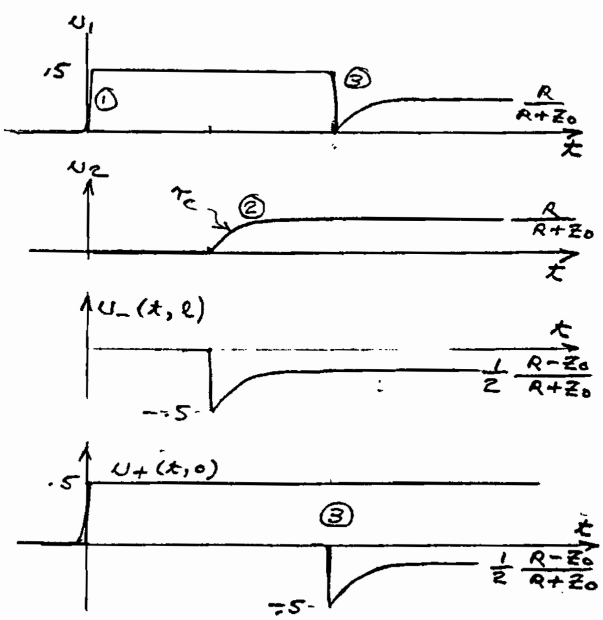


$2v_- = -.33\,u_{-1}(t - 2\tau)$

$v_1 = +.33$ $v_+ = v_1 - v_- = .50$ (NO CHANGE)
OR NOTE $\Gamma_\ell = 0$ SO $\Delta v_+ = 0$

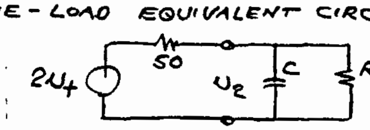
EXAMPLE – ADD SHUNT CAPACITOR TO TERMINATION IN ABOVE CASE





① NO CHANGE FROM ABOVE EXAMPLE UNTIL $t = \tau$

② LINE-LOAD EQUIVALENT CIRCUIT IS:



$2v_+(t,\ell) = u_{-1}(t-\tau)$ FIND v_2

v_2 IS INITIALLY 0 AND MUST BE CHARGED THRU A TIME CONSTANT, $\tau_c = \frac{R \cdot Z_0}{R + Z_0} \cdot C$ TO A FINAL VOLTAGE $= \frac{R}{R+Z_0} \cdot 1$

$$v_2(t) = \frac{R}{R+Z_0} \cdot u_{-1}(t-\tau) \cdot \left(1 - e^{-(t-\tau)/\tau_c}\right)$$

$$v_-(t,\ell) = v_2(t) - v_+(t,\ell)$$
$$= v_2(t) - \frac{1}{2} u_{-1}(t-\tau)$$

③ AT $z = 0$ AND $t = 2\tau$ USE LINE-LOAD CIRCUIT AGAIN OR NOTE THAT v_- IS ABSORBED IN Z_0 AND THAT THERE IS NO CHANGE IN v_+. v_1 IS GIVEN BY $v_+ + v_-$

FOURIER TRANSFORMS OF REFLECTION COEFFICIENTS

ANY OF THE FREQUENCY DOMAIN NETWORK PARAMETERS (I.E. $Z_{11}, S_{11}, S_{21}, A, T_{ik}$, ETC) IS A FOURIER TRANSFORM PAIR WITH THE IMPULSE RESPONSE RELATING THE SAME NETWORK VARIABLES. WE WILL USE THE REFLECTION COEFFICIENT, $\Gamma(f)$ TRANSFORMING TO IMPULSE RESPONSE, $\gamma(t)$ AS A SPECIFIC EXAMPLE AND WILL ASSUME $Z_g = Z_0$

$$\Gamma(f) = \frac{V_-(f)}{V_+(f)} \qquad \Gamma(f) \xleftrightarrow[F.T.]{} \gamma(t) = v_-(t) \quad \text{FOR } v_+(t) = u_0(t) \text{ (IMPULSE)}$$

THE STEP RESPONSE, $g(t) = v_-(t)$ FOR $v_+(t) = u_{-1}(t)$, IS OFTEN COMPUTED OR MEASURED RATHER THAN THE IMPULSE RESPONSE. THE IMPULSE RESPONSE IS SIMPLY THE TIME DERIVATIVE OF $g(t)$

$$\gamma(t) = \frac{d}{dt} g(t) \equiv g'(t) \qquad g(t) = \int_0^t \gamma(t')\,dt'$$

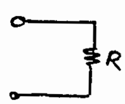
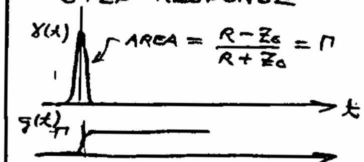
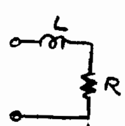
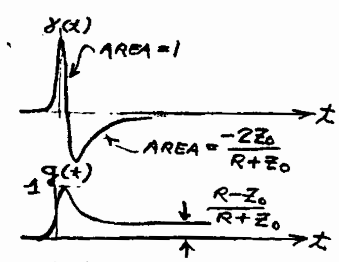
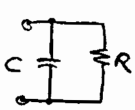
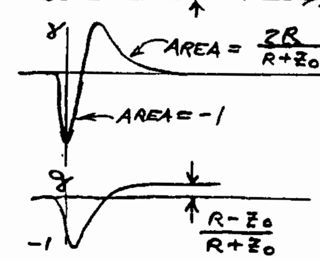
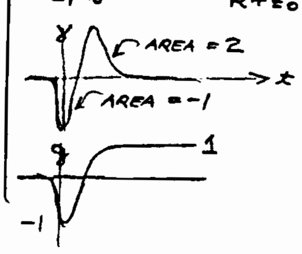
$\gamma(t)$ MUST BE ZERO FOR $t \le 0$ (A NETWORK MAY NOT LAUGH BEFORE IT IS TICKLED!) AND MUST BE REAL. THIS PUTS CONSTRAINTS ON REALIZABLE $\Gamma(f)$ FUNCTIONS SUCH AS NO POLES IN THE RIGHT HALF PLANE AND $\Gamma(-f) = \Gamma^*(f)$. (NO RHP POLES MEANS FOR $\Gamma(f) = \sum \frac{A_k}{j\omega + \alpha_k}$ $\;Re\,\alpha_k \ge 0$)

THE FOURIER TRANSFORMS ARE

$$\Gamma(f) = \int_{-\infty}^{\infty} \gamma(t) e^{-j\omega t}\,dt \qquad \gamma(t) = \int_{-\infty}^{\infty} \Gamma(f) e^{j\omega t}\,df$$

IF $\Gamma(f) = \frac{1}{j\omega + \alpha}$, $\gamma(t) = e^{-\alpha t}$

TRANSFORMS OF REFLECTION COEFFICIENTS

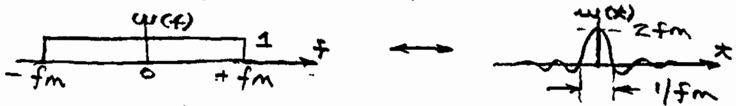
ELEMENT	FREQ DOMAIN, $\Gamma(f)$	TIME DOMAIN, $\gamma(t)$ FOR $t \ge 0$	IMPULSE AND STEP RESPONSE
	$\Gamma(f) = \frac{R - Z_0}{R + Z_0}$	$\gamma(t) = \frac{R - Z_0}{R + Z_0} u_0(t)$	
	$\Gamma(f) = \frac{R + j\omega L - Z_0}{R + j\omega L + Z_0}$ $= 1 - \frac{2Z_0}{L} \cdot \frac{1}{j\omega + \frac{R+Z_0}{L}}$	$\gamma(t) = u_0(t) - \frac{2Z_0}{L} e^{-t/\tau_L}$ $\tau_L = L/(R+Z_0)$	
	$\Gamma(f) = \frac{R - j/\omega C - Z_0}{R - j/\omega C + Z_0}$ $= -1 + \frac{2}{Z_0 C} \cdot \frac{1}{j\omega + \frac{R + Z_0}{Z_0 RC}}$	$\gamma(t) = -u_0(t) + \frac{2}{Z_0 C} e^{-t/\tau_C}$ $\tau_C = \frac{Z_0 RC}{Z_0 + R}$	
	$\Gamma(f) = \frac{-j/\omega C - Z_0}{-j/\omega C + Z_0}$ $= -1 + \frac{2}{Z_0 C} \cdot \frac{1}{j\omega + 1/Z_0 C}$	$\gamma(t) = -u_0(t) + \frac{2}{Z_0 C} e^{-t/\tau_C}$ $\tau_C = Z_0 C$	

APPLICATION OF TRANSFORM PROPERTIES

FREQUENCY WINDOW (OCCURS BECAUSE OF LIMITED MEASUREMENT FREQUENCY RANGE)

IF WE MULTIPLY $\Gamma(f)$ BY A WINDOW FUNCTION $W(f)$ THEN $\gamma(t)$ IS CONVOLVED (SMOOTHED) BY $w(t)$; THE TRANSFORM OF $W(f)$.

EXAMPLE



IF f_m = 26 GHZ $\quad f_m^{-1} = 38.5$ ps (1.16 cm FOR $\epsilon_r = 1$) $\qquad$ HALF-POWER WIDTH ~ $0.6/f_m$

FREQUENCY SAMPLING (OCCURS WHEN VECTOR ANALYSER STEPS IN FREQUENCY)

IF WE MULTIPLY $\Gamma(f)$ BY AN IMPULSE TRAIN SPACED Δf APART THEN $\gamma(t)$ IS CONVOLVED WITH AN IMPULSE TRAIN SPACED $1/\Delta f$. $\gamma(t)$ WILL THEN REPEAT AT $1/\Delta f$ INTERVALS $\gamma(t \pm m/\Delta f) = \gamma(t)$. HENCE IF $\gamma(t)$ IS >0 FOR $t > 1/\Delta f$ THERE MAY BE AN AMBIGUITY PROBLEM.

EXAMPLE $\quad \Delta f = \frac{26 \text{ GHZ}}{200} = 130$ MHZ $\quad \Delta f^{-1} = 7.69$ ns $= 7690$ ps $= 200 \times f_m^{-1}$

TIME DELAY (OCCURS IN A TRANSMISSION LINE)

A TIME SHIFT AS $\gamma(t-\tau)$ MULTIPLIES $\Gamma(f)$ BY $e^{-j\omega\tau}$

THUS ANY OF THE CIRCUIT ELEMENTS ON p. 74 PLACED AT THE END OF A TRANSMISSION LINE OF DELAY τ WILL GIVE REFLECTION COEFFICIENT $\Gamma(f)\, e^{-j\omega 2\tau} \longleftrightarrow \gamma(t-2\tau)$ AT THE INPUT OF THE TRANSMISSION LINE

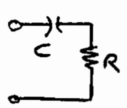
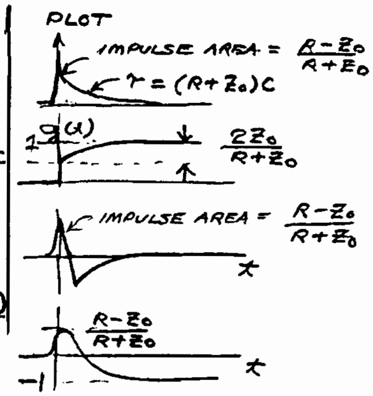
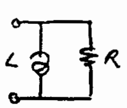
TIME WINDOW (USED TO GATE A PORTION OF IMPULSE RESPONSE CONTAINING DESIRED INFORMATION. GATED IMPULSE RESPONSE IS THEN FOURIER TRANSFORMED TO GIVE FREQUENCY RESPONSE OF SELECTED PORTION OF CIRCUIT)

$$\gamma_g(t) = \gamma(t)\cdot h(t) \qquad \Gamma_g(f) = \Gamma(f) * H(f)$$

IF $h(t)$ IS A RECTANGULAR PULSE OF WIDTH W CENTERED AT t_0

THEN $\quad H(f) = \dfrac{\sin \pi W f}{\pi W f} \cdot e^{-j\omega t_0} \qquad |H(f)|$ HAS WIDTH $= 2/W$

MORE TRANSFORMS

ELEMENT	FREQ DOMAIN, $\Gamma(f)$	TIME DOMAIN, $\gamma(t)$	PLOT
	$\Gamma(f) = \dfrac{R - j/\omega C - Z_0}{R - j/\omega C + Z_0} = \dfrac{R-Z_0}{R+Z_0} + \dfrac{2Z_0}{C(R+Z_0)^2} \cdot \dfrac{1}{j\omega + \tau_c^{-1}}$, $\tau_c = (R+Z_0)C$	$\gamma(t) = \dfrac{R-Z_0}{R+Z_0} u_0(t) + \dfrac{2Z_0}{C(R+Z_0)^2} e^{-t/\tau_c}$, $t \geq 0$, $\tau_c = (R+Z_0)C$	
	$\Gamma(f) = \dfrac{\frac{j\omega LR}{R+j\omega L} - Z_0}{\frac{j\omega LR}{R+j\omega L} + Z_0}$	$\gamma(t) = \dfrac{R-Z_0}{R+Z_0} u_0(t) - \dfrac{2Z_0R^2}{L(R+Z_0)^2} e^{-t/\tau_2}$, $\tau_2 = \dfrac{L(R+Z_0)}{RZ_0}$	

TRANSFORM LIMIT PROPERTIES

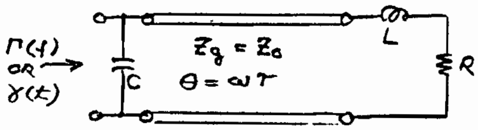
STEP FUNCTION RESPONSE AT $t = 0$ AND ∞ GIVES $\Gamma(f)$ AT $f = \infty$ AND 0

$$g(\infty) = \int_{-\infty}^{\infty} \gamma(t)\, dt = \Gamma(0) \qquad g(0) \stackrel{?}{=} \Gamma(\infty)$$

NO PROOF! BUT TRUE FOR ALL EXAMPLES ABOVE AND 74

EXAMPLE OF A PROBLEM SIMPLIFIED BY A TIME DOMAIN APPROACH

REFERENCE TO OTHER EXAMPLES: HINES AND STINEHELFER, IEEE MTT-22, MARCH, 1974 pp. 276 – 282.



C SMALL SO $Z_0 C \ll \tau$

FREQUENCY DOMAIN APPROACH

LET $z = \dfrac{R + j\omega L}{Z_0}$ $\quad b = \omega C Z_0$

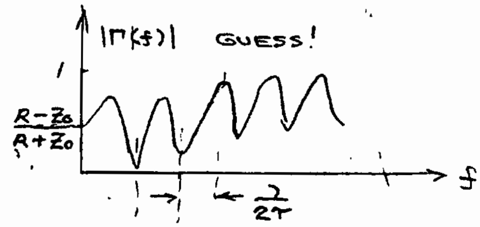
TRANSFORMATION OF z THRU LINE TO GIVE y

$$y = \frac{1 + jz\tan\theta}{z + j\tan\theta} \qquad y_{in} = jb + y$$

$$\Gamma(f) = \frac{1 - y_{in}}{1 + y_{in}} = -1 + \frac{2}{1 + y_{in}} = -1 + \frac{2}{1 + jb + y}$$

$$= -1 + \frac{2 + 2jz\tan\theta}{(1 + jb)(z + j\tan\theta) + 1 + jz\tan\theta}$$

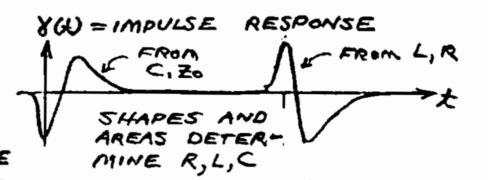
DIFFICULT TO SEPARATE EFFECTS OF L AND C!

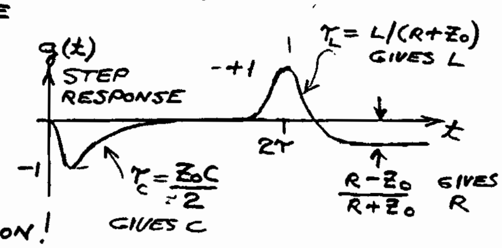


TIME DOMAIN APPROACH

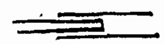
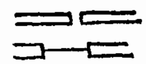
ASSUME τ IS LONG ENOUGH SO THAT THE T-LINE PRESENTS Z_0 DURING CAPACITOR IMPULSE RESPONSE. THUS $\gamma_c(t)$ IS FROM THE TABLE ON p. 74. SMALL C WILL HAVE NEGLIBLE EFFECT ON WAVE TRANSMITTED ON T-LINE TO LOAD. THE REFECTION COEFFICIENT IS FROM 74 WITH TIME SHIFT, τ, AS SHOWN AT RIGHT

THE TIME DELAY BETWEEN RESPONSES OF SEPARATE ELEMENTS ALLOWS EASY INTERPRETATION!

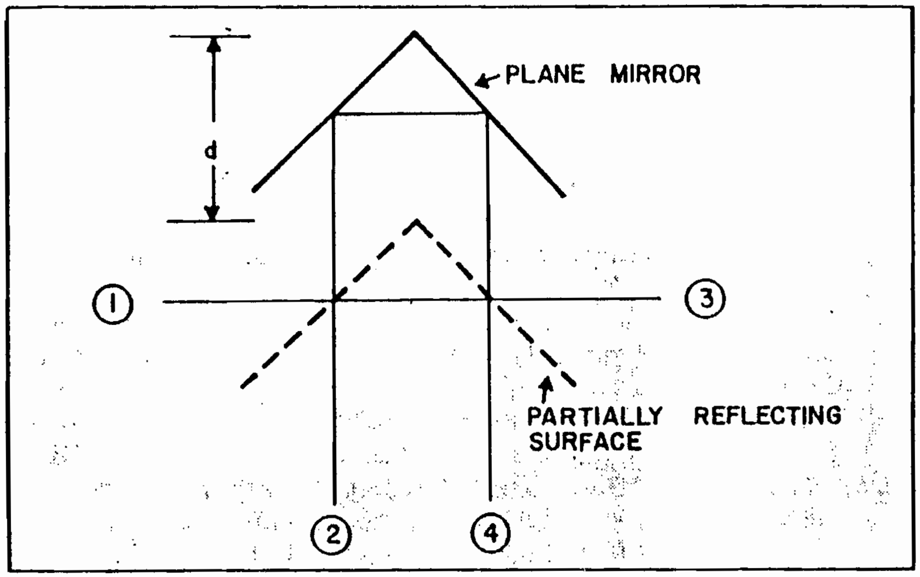




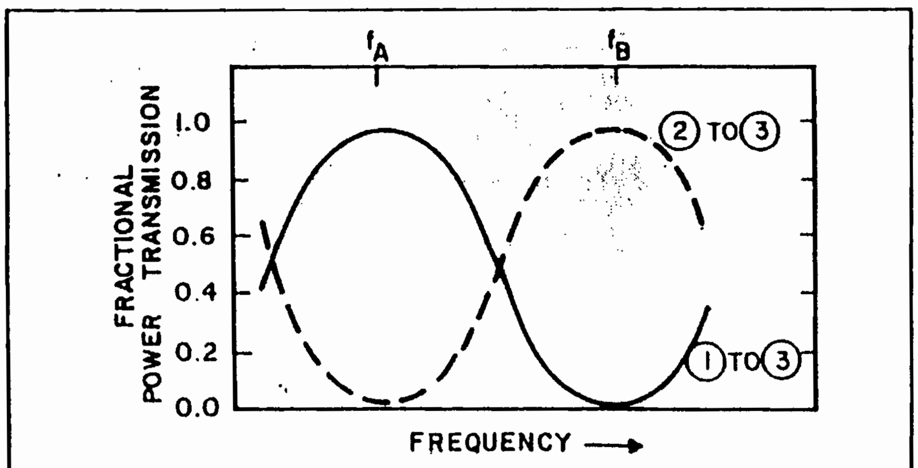
MICROWAVE PASSIVE COMPONENTS SUMMARY

	LUMPED ELEMENTS	COAXIAL	MICROSTRIP OR STRIPLINE	WAVEGUIDE	QUASI-OPTICAL
FREQUENCY RANGE	0 – 2 GHZ	0.1 – 40 GHZ	0.1 – 100 GHZ	1 – 300 GHZ	> 100 GHZ
REFERENCE	ADVANCES IN MICROWAVES VOL 8, M. CAULTON	MICROWAVE TRANSMISSION VOL 9 OF RAD. LAB RAGAN	MICROSTRIP LINES AND SLOT-LINES GUPTA	WAVEGUIDE HANDBOOK VOL 10 OF RAD LAB MARCUVITZ	INFRARED AND MM WAVES BUTTON, VOL 6, CHAPTER 5 BY GOLDSMITH
TYPICAL MFGS	AMERICAN TECH CERAMICS AND MURATA ERIE (CAPACITORS) MANY OTHERS	NARDA, OMNI-SPECTRA, KEVLIN, SAGE, MICROLAB, MIDWEST	ANAREN (STRIPLINE) ROGERS, 3M (DIELECTRIC) TEK-WAVE (FAB) MPC (FAB)	WAVELINE (< 50 GHZ) MACOM HUGHES AEROWAVE } (> 26 GHZ)	MILLITECH
SHUNT REACTANCE	AXIAL LEAD OR CHIP CAPACITORS AND INDUCTORS ≥ 0.1 PF ≥ 10 NH	SHORT-CIRCUITED STUB	OPEN-CIRCUIT STUB SPIRAL INDUCTORS	OBSTACLES – POST, IRIS, SEPTUM	METALLIC MESH OR GRID
SERIES REACTANCE		RE-ENTRANT LINE 	GAP HI-Z LINE SPIRAL INDUCTOR 	TEE 	(NONE)
VARIABLE REACTANCE	PISTON CAPACITORS MOVABLE CORE INDUCTORS	ADJUSTABLE SHORT-CIRCUIT STUB	DIFFICULT (CUT AND TRY)	MOVABLE BACK-SHORTS, TUNING POSTS	ROTATING GRID
FIXED ATTENUATOR	AXIAL RESISTORS < 500MHZ	PI OR TEE RESISTORS IN COAXIAL HOUSING	FILM RESISTOR NETWORKS	LOSSY VANE	LOSSY DIELECTRIC, WEDGE OR CONE SHAPE
VARIABLE ATTENUATOR	POTS < 100 MHZ SWITCHED RESISTORS	SWITCHED PADS PIN DIODES	WIPER ON LOSSY FILM PIN DIODES	MOVING VANE ROTARY VANE CUTOFF ATTENUATOR	
VARIABLE PHASE OR DELAY	SWITCHED NETWORKS	TROMBONE LINE STRETCHER	VARACTOR DIODE	SLIDING SHORT ROTARY DIELECTRIC VARACTOR FERRITE	MOVABLE REFLECTOR PATH
DIRECTIONAL COUPLER	TRANSFORMER + CAPACITORS	COUPLED LINES	COUPLED LINES BRANCH LINES	MULTI-HOLE CROSS GUIDE SIDE-WALL	DIELECTRIC OR GRID AT 45°
SWITCH	MECHANICAL, FET	MECHANICAL, DIODE	DIODE, FET	MECHANICAL FERRITE DIODE	ROTATABLE MIRROR
FILTER	L-C < 500 MHZ	HI-LO Z T-LINE	COUPLED LINES, STUBS	MULTIPLE OBSTACLE WAFFLE-IRON	FABRY-PEROT (CASCADED MESH) MICHELSON (INTERFERENCE)
ISOLATORS AND CIRCULATORS	> 100 MHZ NARROW BAND	USUALLY STRIP-LINE WITH COAXIAL CONNECTORS	STRIP-LINE OCTAVE BANDWIDTH, HIGHLY DEVELOPED	FARADAY ROTATOR Y-JUNCTION	(NONE)
TRANSITIONS	MICROSTRIP – EASY COAXIAL – FAIR WAVEGUIDE – DIFFICULT	MICROSTRIP – EASY STRIPLINE – EASY WAVEGUIDE – EASY	COAXIAL – EASY WAVEGUIDE – FAIR LUMPED – EASY	SIZE CHANGE – EASY FREE SPACE – EASY	WAVEGUIDE THRU HORN MICROSTRIP – FAIR

FROM "QUASI-OPTICAL TECHNIQUES ... ",
P. GOLDSMITH, MIC. SYS. NEWS (MSN)),
DECEMBER, 1983



12a. Basic dual-beam Interferometer *with 4 ports indicated. The path-length difference between the two beams is equal to 2d.*



12b. Transmission function of dual-beam Interferometer. *If there is a mixer receiver at port (3), power will be transmitted very efficiently at frequency A from port, but power at frequency f_B will be severely attenuated, indicating use of this device as a single-sideband filter.*

References

1. F.A. Benson, Ed., "Attenuation of Rectangular Waveguides," *Millimeter and Submillimeter Waves*, Ch. 14, Iliffe, 1969, London
2. R.J. Batt, J.D. Jones, and J.D. Harris, "The Measurement of the Surface Resistivity of Evaporated Gold at 890 GHz," *IEEE Trans. MTT*, Vol. MTT-25, No. 6, June 1977, pp. 488–491.
3. P.T. Parrish, A.G. Cardiasmenos, and I. Galin, "94 GHz Beam-Lead Balanced Mixer," *IEEE Trans. MTT*, Vol. MTT-29, No. 11, November 1981, pp. 1150–1157.
4. P.J. Meier, "Finline Components," *IEEE Trans. MTT*, Vol. MTT-22, No. 12, December 1974, pp. 1209–1216.
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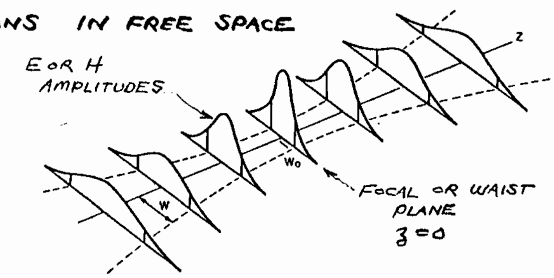
20. N.R. Erickson, "A Directional Filter Diplexer Using Optical Techniques for Millimeter and Submillimeter Wavelengths," *IEEE Trans. MTT*, Vol. MTT-25, No. 10, October 1977, pp. 865–866.

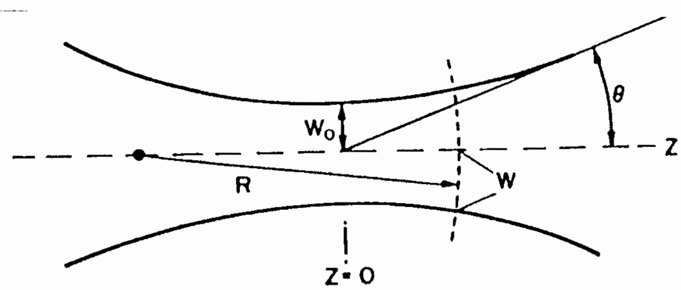
QUASI-OPTIC TECHNIQUES

- PRINCIPLE IS WAVE DIRECTING RATHER THAN WAVE GUIDING AT LONGER WAVELENGTHS OR CURRENT GUIDING AT STILL LONGER λ
- USED AT WAVELENGTHS TYPICALLY BETWEEN 3 mm AND 0.1 mm (100 GHZ TO 3000 GHZ) WHERE WAVEGUIDES ARE INCONVENIENTLY SMALL AND LOSSY
- DISTINGUISHED FROM GEOMETRICAL OR "PURE" OPTICS BY CONSIDERATION OF NON-ZERO WAVELENGTH, DIFFRACTION EFFECTS, AND USE OF METALLIC ELEMENTS SUCH AS GRIDS AND MESHES.
- TYPICAL SIZE OF WAVE DIRECTING ELEMENTS IS 30 TO 300 λ COMPARED TO $>1000\,\lambda$ FOR THE OPTICAL RANGE.
- GEOMETRICAL OPTICS ALLOWS WAVES TO BE FOCUSED TO A POINT WHEREAS QUASI-OPTICS CONSIDERS THAT FOCUS IS TO A MINIMUM SIZE AREA CALLED THE BEAM WAIST OF RADIUS, W_0.
- PRIME ANALYTICAL TECHNIQUE IS THE GAUSSIAN BEAM MODE SOLUTION TO MAXWELLS EQUATIONS IN FREE SPACE



The profile of a beam-mode with Gaussian amplitude distribution.

GAUSSIAN BEAM MODES



The parameters w, R and θ that characterise a Gaussian beam-mode.

DOMINANT MODE SOLUTION FOR E OR H IS OF FORM

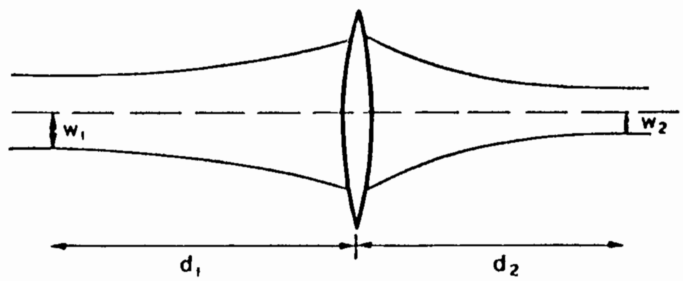
$$E = E_0 \frac{W_0}{W_1} \underbrace{\exp\left(\frac{-r^2}{w^2}\right)}_{\text{GAUSSIAN } r \text{ DEPENDENCE}} \cdot \underbrace{\exp\left[-j(\beta z-\phi)\right]}_{\text{PHASE VARIATION IN } z} \cdot \underbrace{\exp\left(\frac{-j\beta r^2}{2R}\right)}_{\text{PHASE FRONT CURVATURE}}$$

WHERE $W(z) = W_0\left[1+\left(\frac{\lambda z}{\pi W_0^2}\right)^2\right]^{1/2}$ $\equiv$ WAIST AS FUNCTION OF z

$R(z) = z\left[1+\left(\frac{\pi W_0^2}{\lambda z}\right)^2\right]$ $\equiv$ RADIUS OF CURVATURE

$\phi(z) = \tan^{-1}\left(\frac{\lambda z}{\pi W_0^2}\right)$ $\equiv$ ADDITIONAL PHASE SHIFT $-\frac{\pi}{2} \le \phi \le \frac{\pi}{2}$

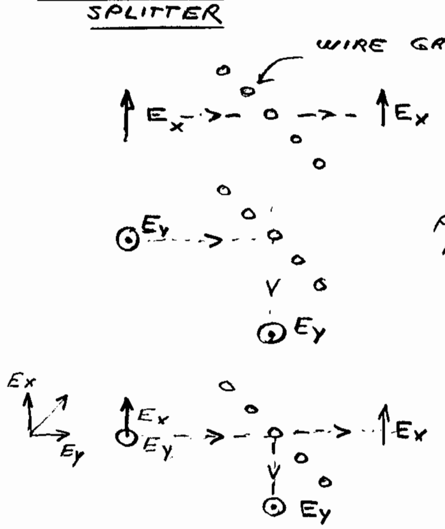
- HIGHER ORDER GAUSSIAN MODES CONSIST OF THE ABOVE MULTIPLIED BY LAGUERRE POLYNOMIALS.
- AN EXCELLENT TUTORIAL ON GAUSSIAN MODES IS GIVEN BY KOGELNIK AND LI, PROC. IEEE, VOL 54, p. 1322-1329, OCT 1966
- AN EXCELLENT INTRODUCTION TO QUASI-OPTICAL TECHNIQUES IS GIVEN BY GOLDSMITH, MIC. SYS. NEWS, pp 65-84, IN APPENDIX F
- LENS AND CURVED REFLECTORS CAN BE USED TO CHANGE THE WAIST, w_0, OF A GAUSSIAN MODE (SEE APPENDIX F)



A lens employed to convert a Gaussian beam from one waist size and position (w_1, d_1) to another (w_2, d_2).

REFLECTORS SUCH AS PARABOLOIDS AND ELLIPSOIDS CAN ALSO RE-FOCUS BEAMS AND ALSO RE-DIRECT THE BEAM WITH LOWER LOSS THAN REFRACTIVE ELEMENTS

WIRE-GRID POLARIZED POWER SPLITTER



PERPENDICULAR E IS TRANSMITTED THRU GRID

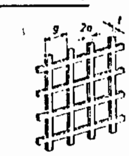
PARALLEL E IS REFLECTED

45° E IS 1/2 REFLECTED AND 1/2 TRANSMITTED

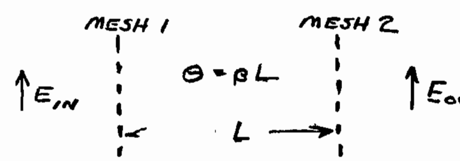
SEE MARTIN AND PUPLETT, J. OF INFRARED PHYSICS VOL 10, pp. 105-109, 1969

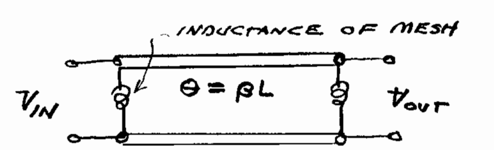
FABRY-PEROT BANDPASS FILTER

MESH (OR DIELECTRIC) IS USED AS OBSTACLE OR SHUNT REACTANCE



SEE ULRICH, J. OF INFRARED PHYSICS, VOL 7, pp. 37-55, 1967





EQUIVALENT T-LINE CIRCUIT

L CAN BE ADJUSTED FOR PARALELL RESONCE

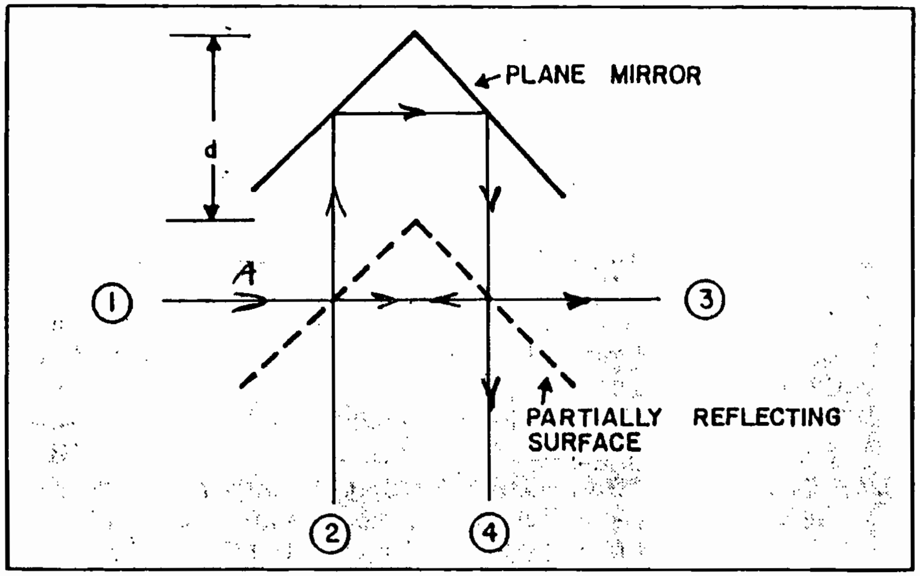
DUAL BEAM MICHELSON INTERFEROMETER OR Q-O DIRECTIONAL FILTER

(SEE DRAWING ON NEXT PAGE)

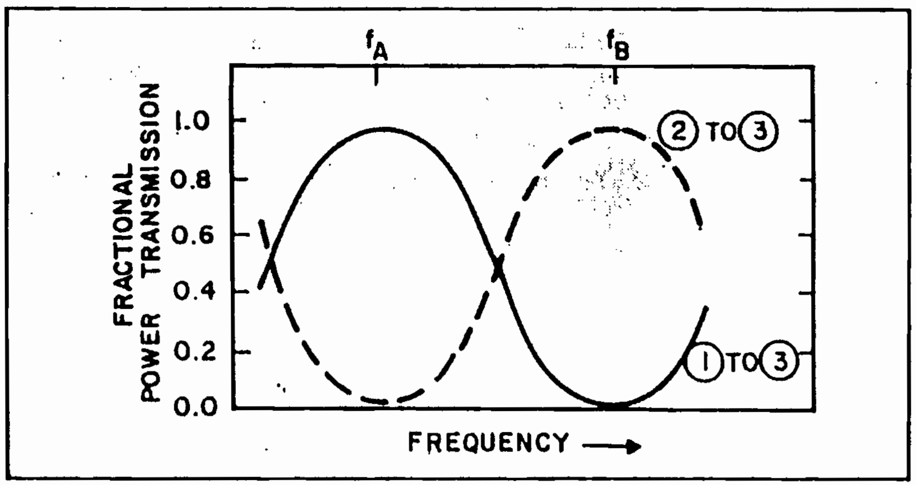
AN INCOMING BEAM ① IS SPLIT INTO TWO PATHS BY POWER SPLITTER A

THE TWO BEAMS RECOMBINE AT POWER SPLITTER B WITH DIRECTION TO ③ OR ④ DEPENDENT UPON THE PATH LENGTH DIFFERENCE AND FREQUENCY

FROM "QUASI-OPTICAL TECHNIQUES ... ", P. GOLDSMITH, MIC. SYS. NEWS (MSN), DECEMBER, 1983



12a. Basic dual-beam interferometer *with 4 ports indicated. The path-length difference between the two beams is equal to 2d.*



12b. Transmission function of dual-beam interferometer. *If there is a mixer receiver at port (3), power will be transmitted very efficiently at frequency A from port, but power at frequency f_B will be severely attenuated, indicating use of this device as a single-sideband filter.*

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3. P.T. Parrish, A.G. Cardiasmenos, and I. Galin, "94 GHz Beam-Lead Balanced Mixer," *IEEE Trans. MTT,* Vol. MTT-29, No. 11, November 1981, pp. 1150–1157.
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12. R.H. Garnham, "Quasi-Optical Components," *Millimeter and Submillimeter Waves,* F.A. Benson, Ed., Ch. 21, Iliffe, 1969, London
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17. P.F. Goldsmith, "Diffraction Loss in Dielectric-Filled Fabry-Perot Interferometers," *IEEE Trans. MTT,* Vol. MTT-30, No. 5, May 1982, pp. 820–823.
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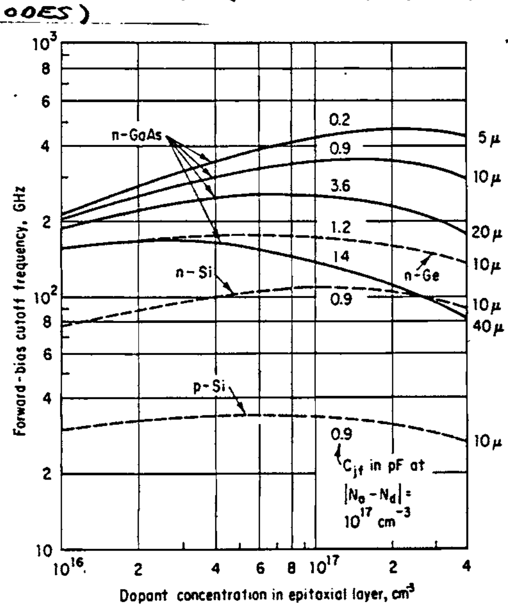
MICROWAVE DIODES

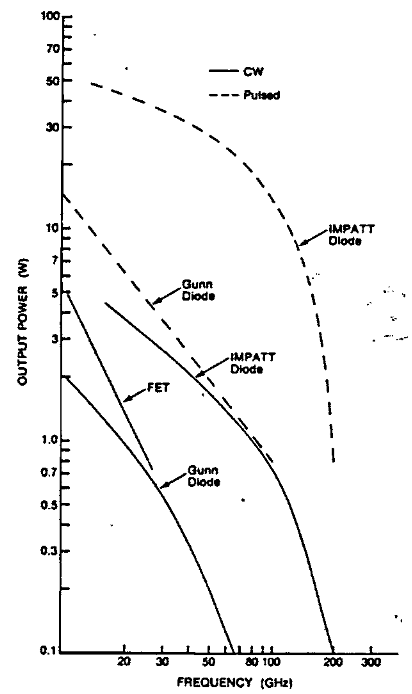
TYPE	MATERIAL	APPLICATION	CRITICAL CHARACTERISTICS
SCHOTTKY	SI, GAAS	DETECTORS MIXERS	SENSITIVITY, SQUARE-LAW CONVERSION LOSS, NOISE
VARACTOR	SI, GAAS	TUNING PHASE SHIFTERS FREQ. MULTIPLIERS PARAMPS	C_{MAX}/C_{MIN}, Q " , Q EFFICIENCY, P_{OUT} HIGH f_{CO}
PIN	SILICON	SWITCHING ATTENUATION	LOW R_{ON}, C LOW C
GUNN	GAAS, INP	OSCILLATOR	P_{OUT} VS F
IMPATT	SI, GAAS	OSCILLATOR	P_{OUT} VS F, NOISE
TUNNEL	INSB	AMPLIFIERS	NEGATIVE RESISTANCE, SHOT NOISE

CUTOFF FREQUENCY $f_{CO} = \frac{1}{2\pi R_S C_0}$ OF SCHOTTKY DIODES AS A FUNCTION OF MATERIAL, DOPING AND DIAMETER (FOR MICROWAVE DIODES)



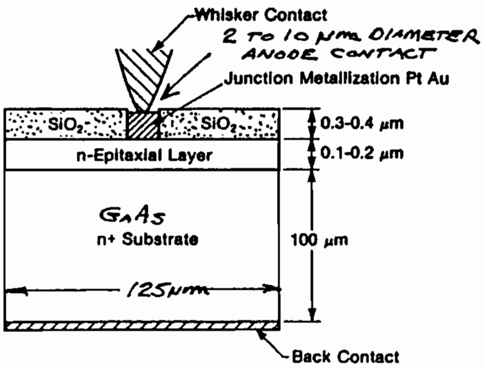
FROM WATSON, MICROWAVE SEMICONDUCTOR DEVICES AND THEIR CIRCUIT APPLICATIONS, McGRAW HILL, 1969.

POWER OUTPUT VS FREQUENCY FOR FET, GUNN, AND IMPATT OSCILLATORS

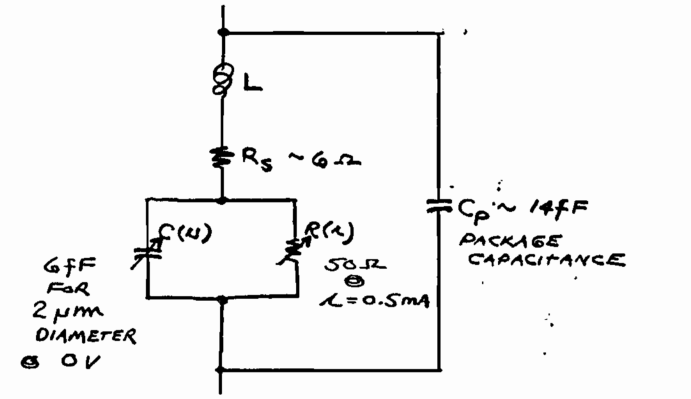


FROM BHARTIA AND BAHL, MMλ ENGINEERING AND APPLICATIONS, WILEY, 1984

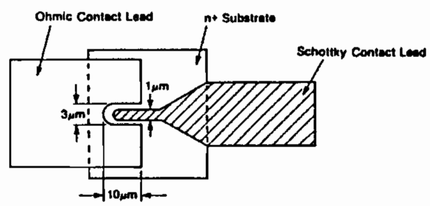
MILLIMETER WAVE SCHOTTKY DIODES



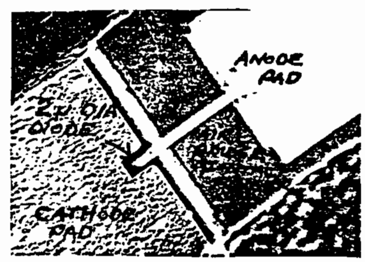
SCHOTTKY, HONEYCOMB DIODE CHIP (NOT TO SCALE)



DIODE EQUIVALENT CIRCUIT WITH TYPICAL VALUES FOR 2μm ANODE DIAMETER



PLANER BEAM-LEAD DIODE



SEM PHOTOGRAPH OF UVA PLANER DIODE

SCHOTTKY DIODE CHARACTERISTICS

$$i = i_s\left(e^{qv/\eta kT} - 1\right)$$

$$g(i) \equiv \frac{di}{dv} = \frac{q}{\eta kT}(i + i_s)$$

$$R(i) = \frac{\eta kT}{q(i+i_s)} \sim \frac{25}{i\,(\text{mA})} \quad @\ T = 290K$$

$$C(v) = C_0 / \sqrt{1 - v/\phi} \qquad \phi = \text{DIFFUSION POTENTIAL} \sim 0.9 \text{ VOLTS}$$

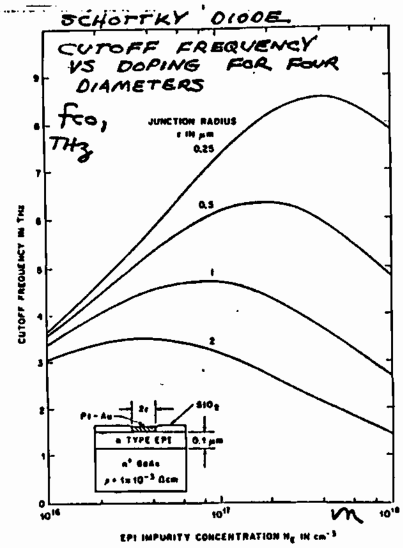
$$C_0 \propto m^{1/2} D^2 \qquad R_s \propto m^{-1} D^{-1} \quad \text{FOR LARGE DIODES}$$

$$f_{co} = \frac{1}{2\pi R_s C_0} = \text{CUTOFF FREQUENCY, SEE PLOT AT RIGHT}$$

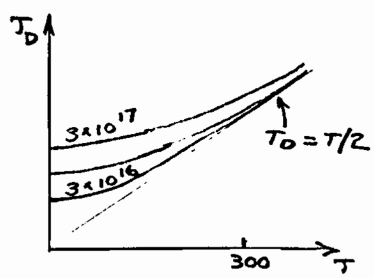
NOISE TEMPERATURE T_D, OF $R(i)$ IS $\sim T/2$ FOR $T \sim 300K$. AS DIODE IS COOLED T_D APPROACHES A CONSTANT VALUE $\propto m$ (DOPING DENSITY). THIS FAVORS LOW DOPED DIODES FOR CRYOGENIC OPERATION

REF: VIOLA AND MATTAUCH, J. OF AP. PHY, 44, 1973, p. 2805

MILLIMETER WAVE SCHOTTKY DIODE

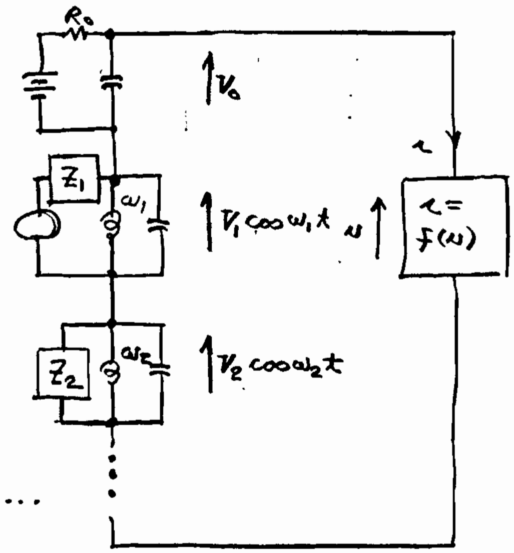


(FROM M. SCHNEIDER, BTL)



GENERAL NON-LINEAR CIRCUIT ANALYSIS BY POWER SERIES EXPANSION

CIRCUIT IS DESIGNED TO BE IN FORM SHOWN BELOW OR ITS DUAL WITH $v = f(i)$. THE IMPORTANT CHARACTERISTIC IS THAT THE FREQUENCIES THAT MAY EXIST IN THE INDEPENDENT VARIABLE (v) ARE LIMITED BY FILTERS IN THE CIRCUIT WHICH ALSO SEPARATE THE CURRENTS AT FREQUENCY ω_K INTO A SPECIFIC LOAD Z_K OR SOURCE.



ANALYSIS PROCEDURE

1) SEPARATE INDEPENDENT VARIABLE (v) INTO LARGE AND SMALL SIGNAL PORTIONS AND NOTE WHAT FREQUENCIES ARE ALLOWED IN EACH

$$v = v_p + \delta v$$

2) EXPAND THE NON-LINEARITY IN A POWER SERIES AROUND v_p

$$i = f(v_p) + f'(v_p)\cdot\delta v + \frac{f''(v_p)}{2}\cdot(\delta v)^2 + \cdots$$

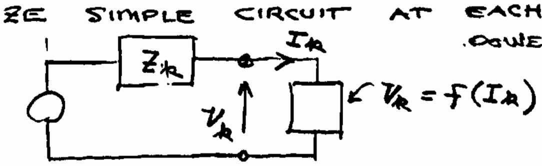
$$f'(v_p) \equiv \left.\frac{di}{dv}\right|_{v=v_p} \equiv \text{TIME VARYING CONDUCTANCE} = g(t)$$

3) WRITE FOURIER SERIES FOR v AND i

$$v = \sum_{k=-K}^{K} V_k e^{j\omega_k t} \qquad i = \sum_{k=-\infty}^{\infty} I_k e^{j\omega_k t}$$

AND FIND I_k'S AS FUNCTION OF V_k'S

4) ANALYZE SIMPLE CIRCUIT AT EACH FREQUENCY TO FIND POWER FLOW AND EQUIVALENT LOAD IMPEDENCES



5) CONSIDER CONSEQUENCES OF APPROXIMATIONS!

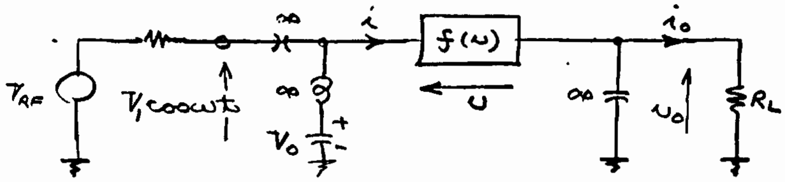
ABOVE METHOD CAN BE APPLIED TO THE FOLLOWING DEVICES:

DEVICE	LARGE SIGNAL	SMALL SIGNAL	IDEAL NON-LINEARITY	REFERENCES
DETECTOR	DC	RF	$I = V^2$	COWLEY AND SORENSON, MTT-14, NO 12, DEC, 1966 PP 588-602
MIXER	DC, PUMP	RF = ω_{RF} IF = $\omega_p - \omega_{RF}$ IMAGE = $2\omega_p - \omega_{RF}$	OPEN/SHORT SWITCH	TORREY AND WHITMER, M.I.T. RAD LAB SALEH, THEORY OF RESISTIVE MIXERS HELD AND KERR, MTT-26?2, FEB 1978
FREQUENCY MULTIPLIERS	DC, ω_1	$\omega_N = N\omega_1$	$Q = V^N$	PENFIELD AND RAFUSE, VARACTOR APPLICATIONS
PARAMETRIC AMPLIFIERS	DC, PUMP	RF = ω_{RF} IDLERS $\omega_p \pm \omega_{RF}$	$Q = V^2$	AS ABOVE

(93)

MICROWAVE DETECTORS

CONSIDER GENERAL CASE OF A BIASED NON-LINEAR DEVICE WITH $i = f(v)$ IN A CIRCUIT AS FOLLOWS:



EXPAND $i = f(v)$ IN A POWER SERIES AROUND V_0

$$i = f^{(0)} + f^{(1)} (v - V_0) + \frac{f^{(2)}}{2} (v - V_0)^2 + \cdots$$

WHERE $f^{(m)} = \left. \frac{d^m f}{dv^m} \right|_{v = V_0}$ ≡ m'th DERIVATIVE EVALUATED AT THE BIAS POINT

THE VOLTAGE IS OF THE FORM:

$$v = V_0 + V_1 \cos\omega_1 t - .$$

(HARMONIC VOLTAGES WHICH COULD EXIST AT POINT A IN THE CIRCUIT ARE NEGLECTED OR COULD BE MADE EQUAL TO ZERO WITH A LOW-PASS FILTER)

EXPRESS CURRENT AS FOURIER SERIES

$$i = I_0 + I_1 \cos\omega_1 t + I_2 \cos 2\omega_1 t + \cdots$$

ONLY COS TERMS SINCE COS EXCITATION OF RESISTIVE NETWORK

(94)

EQUATING COEFFICIENTS GIVES

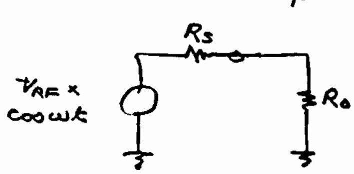
$$I_0 = \underbrace{f^{(0)}}_{\text{DC BIAS}} - \underbrace{f^{(1)} v_0}_{\text{VIDEO CONDUCTANCE}} + \underbrace{\frac{f^{(2)} V_1^2}{4}}_{\text{DESIRED OUTPUT}} + \underbrace{\frac{f^{(2)} v_0^2}{2}}_{\text{SMALL NON LINEARITY}} + \cdots$$

$$I_1 = \underbrace{f^{(1)} V_1}_{\text{RF INPUT CONDUCTANCE}} + \underbrace{\frac{f^{(2)}}{2} (-2 v_0 V_1)}_{\text{NON-LINEAR INPUT CONDUCTANCE}}$$

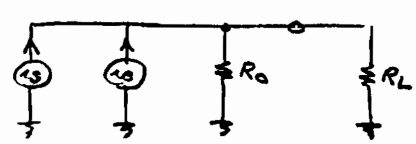
DEFINING: $R_0 \equiv 1/f^{(1)}$ ≡ SMALL-SIGNAL RESISTANCE

$P_1 = \frac{V_1^2}{2 R_0}$ = ABSORBED RF POWER

$\beta = f^{(2)}/2 f^{(1)}$ ≡ CURRENT RESPONSIVITY



RF EQUIVALENT CIRCUIT



DC EQUIVALENT CIRCUIT

$i_S = \beta P_1$ DETECTED CURRENT $i_0 = f^{(0)}$ BIAS CURRENT

APPLICATION TO SCHOTTKY DIODE DETECTOR 95

$$i = i_s\left[e^{qv/\eta kT} - 1\right]$$

$$f^{(0)} = i_s\left[e^{qV_0/\eta kT} - 1\right] = i_B \text{ (BIAS CURRENT)} \qquad f^{(1)} = \frac{q}{\eta kT}(i_B + i_s) \sim \frac{i_B}{.025} = R_0^{-1}$$

$$f^{(2)} = \left(\frac{q}{\eta kT}\right)^2 (i_B + i_s) \qquad \beta = f^{(2)}/2f^{(1)} = \frac{q}{2\eta kT} = \underset{T=290}{\overset{\theta}{20}} \text{ VOLTS}^{-1} \text{ OR } \mu A/\mu W$$

SENSITIVITY – NOISE EQUIVALENT POWER (NEP)

NEP ≡ INPUT POWER THAT PRODUCES DC OUTPUT CURRENT EQUAL TO RMS CURRENT NOISE IN DETECTOR OUTPUT, i_N, IN 1 HZ BAND

IN TERMS OF NOISE TEMPERATURE OF VIDEO RESISTANCE

$$i_N^2 = 4kT_N/R_0 \qquad NEP = \frac{i_N}{\beta} = \sqrt{\frac{4kT_N}{R_0}} \cdot \frac{2f^{(1)}}{f^{(2)}}$$

FOR SCHOTTKY DIODE:

$$R_0 = \frac{\eta kT}{q(i_B + i_s)} \qquad \beta = \frac{q}{2\eta kT} \qquad NEP = \sqrt{\frac{4kT_N q i_B}{\eta kT}} \; \frac{2\eta kT}{q} = 4k\sqrt{\eta T T_N \frac{i_B}{q}}$$

T	T_N	i_B mA	NEP dBm/$\sqrt{Hz}$	NEP watts/$\sqrt{Hz}$	COMMENT
290	145	.02	−99	1.26×10^{-13}	VERY IDEAL CASE, NO 1/f NOISE IN DIODE, $R_0 = 1250$, DIFFICULT TO MATCH
290	10^7	.50	−67.8	1.66×10^{-10}	MORE REALISTIC 1/f NOISE AND $R_0 = 50$ OHMS

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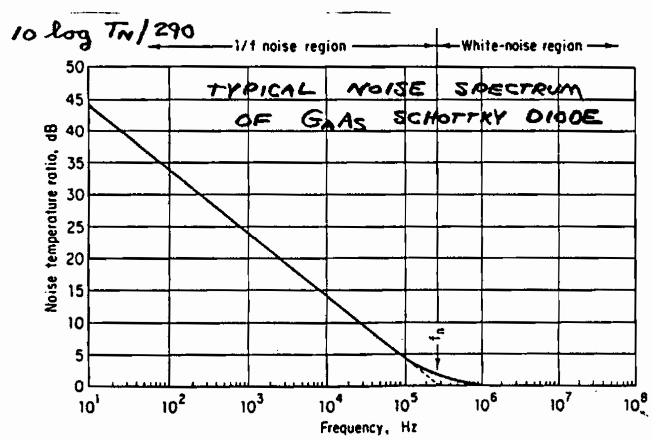
ANOTHER SENSITIVITY PARAMETER IS TANGENTIAL SENSITIVITY (TSS) IN 2 MHZ VIDEO BANDWIDTH. THIS IS 4 DB (PEAK RATHER THAN RMS NOISE IN DEFINITION) + $10 \log \sqrt{2 \times 10^6}$ =: 4 + 31.5 DB = 35.5 dB ABOVE NEP. TYPICAL VALUE OF TSS IS −50 DBm WHICH GIVES NEP = −85.5 DBm. THE NOISE IN 2MHZ VIDEO BAND IS LESS EFFECTED BY ~~VIDEO BANDWIDTH~~ 1/f NOISE.

NEP OF BEST CRYOGENICALLY COOLED MILLIMETER WAVE (300 GHZ) COOLED DOPED GERMANIUM BOLOMETERS IS ~ 10^{-15} WATTS/$\sqrt{HZ}$



FROM WATSON, MICROWAVE SEMICONDUCTOR DEVICES ..., 1969

APPLICATION TO SCHOTTKY DIODE DETECTOR

$$i = i_s\left[e^{qv/\eta kT} - 1\right]$$

$$f^{(0)} = i_s\left[e^{qV_0/\eta kT} - 1\right] = i_B \text{ (BIAS CURRENT)} \qquad f^{(1)} = \frac{q}{\eta kT}(i_B + i_s) \sim \frac{i_B}{.025} = R_0^{-1}$$

$$f^{(2)} = \left(\frac{q}{\eta kT}\right)^2 (i_B + i_s) \qquad \beta = f^{(2)}/2f^{(1)} = \frac{q}{2\eta kT} \underset{T=290}{\overset{\theta}{=}} 20 \text{ VOLTS}^{-1} \text{ OR } \mu A/\mu W$$

SENSITIVITY – NOISE EQUIVALENT POWER (NEP)

NEP ≡ INPUT POWER THAT PRODUCES DC OUTPUT CURRENT EQUAL TO RMS CURRENT NOISE IN DETECTOR OUTPUT, i_N, IN 1 HZ BAND

IN TERMS OF NOISE TEMPERATURE OF VIDEO RESISTANCE

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FOR SCHOTTKY DIODE:

$$R_0 = \frac{\eta kT}{q(i_B + i_s)} \qquad \beta = \frac{q}{2\eta kT} \qquad NEP = \sqrt{\frac{4kT_N\, q\, i_B}{\eta kT}} \cdot \frac{2\eta kT}{q} = 4k\sqrt{\eta T T_N \frac{i_B}{q}}$$

T	T_N	i_B mA	NEP DBM/$\sqrt{HZ}$	NEP WATTS/$\sqrt{HZ}$	COMMENT
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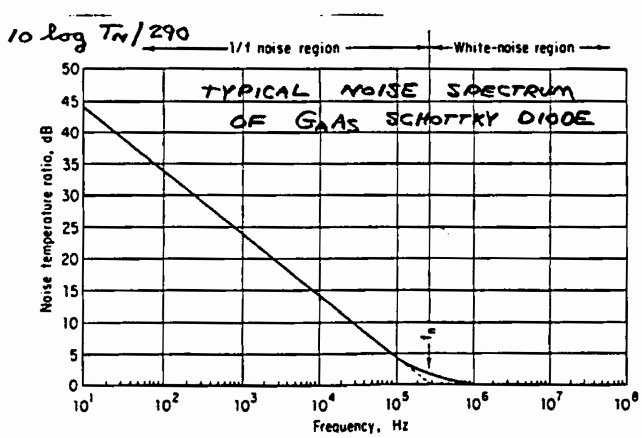


FROM WATSON, MICROWAVE SEMICONDUCTOR DEVICES ···, 1969

MIXERS – THE DIODE RESISTIVE MIXER

APPLY THE POWER SERIES NON-LINEAR ANALYSIS METHOD OF p (91) TO AN IDEAL SCHOTTKY DIODE PUMPED WITH A SINUSOIDAL VOLTAGE AT F

FOLLOWING NOTATION OF TORREY AND WHITMER AND SALEH:

$\omega_p \equiv$ PUMP OR LO FREQUENCY

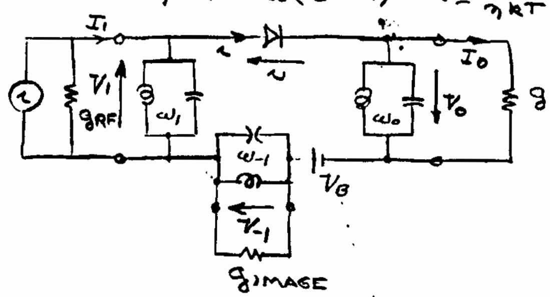
$\omega_0 \equiv$ I.F. FREQUENCY

$\omega_k \equiv k\omega_p + \omega_0$

$\omega_1 \equiv \omega_p + \omega_0 \equiv$ RF FREQUENCY

$\omega_{-1} \equiv \omega_0 - \omega_p \equiv$ IMAGE FREQUENCY

$$i = f(v) = i_s(e^{\alpha v} - 1) \qquad \alpha \equiv \frac{q}{\eta k T}$$



Y-MIXER IDEAL CIRCUIT MODEL

1) LARGE SIGNAL IS $V_B + V_p \cos\omega_p t$

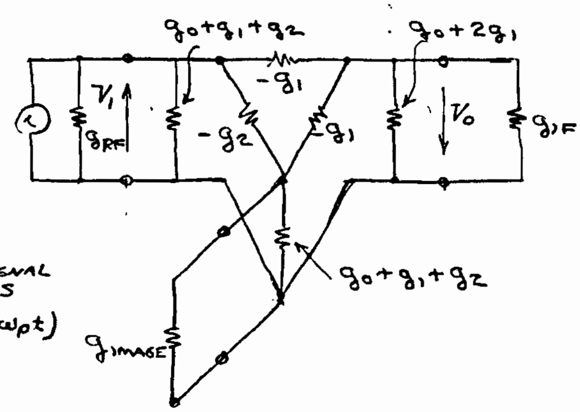
SMALL SIGNAL IS AT ALL ω_k BUT FILTERS RESTRICT VOLTAGE TO FREQUENCIES $\omega_{-1}, \omega_0, \omega_1$

2) NONLINEAR EXPANSION CAN BE TRUNCATED AFTER FIRST DERIVATIVE SINCE $V_p >> V_{RF}$

$$i = i(V_B + V_p) + i'(V_B + V_p) \times \text{SMALL SIGNAL VOLTAGES}$$

$$i'(V_B + V_p) = \left.\frac{di}{dv}\right|_{v = V_B + V_p} = \alpha i_s e^{\alpha(V_B + V_p \cos\omega_p t)}$$

$$\equiv g(t) \equiv \text{TIME-VARYING CONDUCTANCE}$$



Y-MIXER LINEAR EQUIVALENT CIRCUIT (IGNORE FREQUENCY)

THE EXPONENTIAL HAS A FOURIER EXPANSION IN TERMS OF MODIFIED BESSEL FUNCTIONS, $I_k(\alpha V_p)$ SHOWN AT RIGHT

$$g(t) = \alpha i_s e^{\alpha V_B} \sum_{k=-\infty}^{\infty} I_k(\alpha V_p) e^{jk\omega_p t}$$

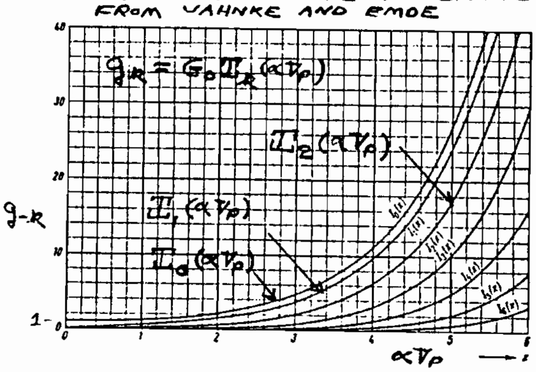
$$= \sum_{k=-\infty}^{\infty} g_k e^{jk\omega_p t}$$

WHERE:

$$g_k = G_0 I_k(\alpha V_p) = g_{-k}$$

$$G_0 = \alpha i_s e^{\alpha V_B}$$

MODIFIED BESSEL FUNCTIONS FROM JAHNKE AND EMDE



THE SMALL SIGNAL EQUATION IS

$$\delta i = g(t) \cdot \delta v$$

$$\delta v = \mathrm{Re} \sum_{k=-1}^{1} V_k e^{j(k\omega_p + \omega_0)t}$$

$$\delta i = \mathrm{Re} \sum_{k=-\infty}^{\infty} I_k e^{j(k\omega_p + \omega_0)t}$$

DC BIAS CURRENT IS GIVEN BY

$$i_{DC} = \overline{i(V_B + V_p)} = i_s e^{\alpha V_B} I_0(\alpha V_p)$$

$$G_0 = \alpha I_{DC} / I_0(\alpha V_p)$$

EQUATING FOURIER COEFFICIENTS GIVES: $\quad I_j = \sum_{k=-1}^{1} g_{k-j} V_k$

		RF	IF	IMAGE
RF	$I_1 =$	$g_0 V_1 +$	$g_1 V_0 +$	$g_2 V_{-1}$
IF	$I_0 =$	$g_1 V_1 +$	$g_0 V_0 +$	$g_1 V_{-1}$
IMAGE	$I_{-1} =$	$g_2 V_1 +$	$g_1 V_0 +$	$g_0 V_{-1}$

THESE EQUATIONS SUGGEST EQUIVALENT RESISTOR NETWORK SHOWN ABOVE

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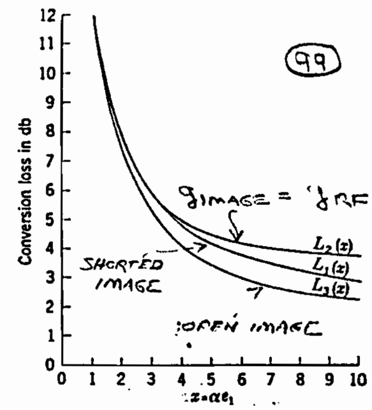
CONVERSION LOSS OF RESISTIVE MIXER

THE CONVERSION LOSS (>1) OF A MIXER

$$L \equiv \frac{\text{RF POWER AVAILABLE FROM SOURCE}}{\text{IF POWER AVAILABLE FROM MIXER}}$$

WITH THE RF SOURCE IMPEDENCE ADJUSTED TO MINIMIZE L. IT IS A FUNCTION OF IMAGE IMPEDANCE AND LO VOLTAGE AS SHOWN AT RIGHT

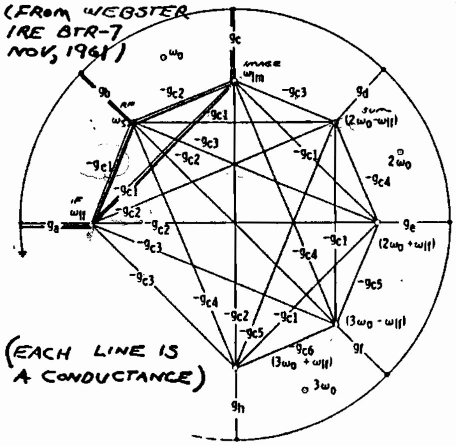


REAL MIXER MODIFICATIONS TO THEORY

1) NON-SINUSOIDAL VOLTAGE PUMP – THE LO VOLTAGE WAVEFORM MUST BE FOUND IN A CIRCUIT WHERE THE LO VOLTAGE SOURCE HAS A SPECIFIED IMPEDANCE. THIS CAN BE DONE WITH NUMERICAL METHODS AND RESULTS IN MODIFICATION OF THE CONDUCTANCE FOURIER COEFFICIENTS

2) VOLTAGES AT SIDEBANDS OF PUMP HARMONICS MAY NOT BE ZERO. IF 2ND AND 3RD HARMONIC SIDEBANDS ARE NOT SHORT-CIRCUITED BY FILTERS THE EQUIVALENT CIRCUIT AT RIGHT RESULTS. POWER IS LOST AT THESE HIGHER FREQUENCIES. NOTE THAT THE CONVERSION TRANSCONDUCTANCE TO THE SUM FREQUENCY, $2\omega_p - \omega_o$, IS EQUAL TO THE RF TO IF VALUE, g_1.

(FROM WEBSTER IRE BTR-7 NOV, 1961)



(EACH LINE IS A CONDUCTANCE)

EQUIVALENT CIRCUIT OF A MIXER INCLUDING 2ND AND 3RD HARMONIC SIDEBANDS. HEAVY LINES WERE INCLUDED IN PREVIOUS ANALYSIS.

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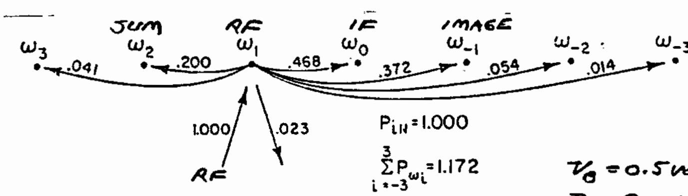
3) THE DIODE SERIES RESTANCE R_s AND JUNCTION CAPACITY INCREASE THE CONVERSION LOSS. AN APPROXIMATE DEGRADATION FACTOR D (WHICH MULTIPLIES CONVERSION LOSS L) IS GIVEN BY SHARPLESS (BSTJ, VOL 35, pp 1385-1402, NOV 1956) AS:

$$D = 1 + 2 f_{RF}/f_c \qquad \text{WHERE} \quad f_c = 1/(2\pi R_s C_o)$$

4) THE JUNCTION CAPACITY, C_j IS NON-LINEAR AND CAN ADD OR SUBTRACT TO THE NON-LINEAR RESISTIVE CONVERSION PROCESS. NOTE THAT A RESISTIVE MIXER IS RECIPROCAL SINCE IT IS REPRESENTED BY A RESISTOR NETWORK. THE CONVERSION LOSS FROM IF TO RF IS THE SAME AS RF TO IF. THIS IS NOT TRUE IF NONLINEAR CAPACITANCE EFFECTS ARE INCLUDED.

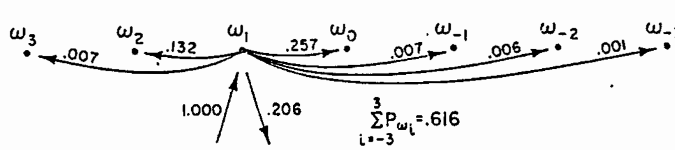
AN ACCURATE ANALYSIS, INCLUDING THE ABOVE EFFECTS, OF AN 115 GHZ SCHOTTKY-DIODE MIXER HAS BEEN PERFORMED BY KERR AND HELD (SEE NASA PUBLICATION X-130-77-6, JANUARY 1977). SOME RESULTS CONCERNING POWER FLOW ARE SHOWN AT RIGHT FOR TWO DIFFERENT POSITIONS OF THE WAVEGUIDE BACKSHORT.



$V_0 = 0.5$ VOLT

$I_0 = 2$ mA

MINIMUM CONVERSION LOSS, L



MINIMUM T_m

MIXER NOISE TEMPERATURE DEFINITIONS 101

The noise temperature description of a mixer is confused because of the presence of an image. In particular, it is unclear as to whether noise generated in the image termination (which, in most cases, is the antenna) is to be included as part of mixer noise, the antenna noise, or as a separate quantity. We will take the latter approach and will derive a consistent set of equations which are valid for mixers with or without image rejection and for arbitrary physical temperatures of the source resistance and mixer diode.

The noise temperature, T_{12}, of a cascade of two networks having noise temperatures T_1 and T_2 is given by the noise figure cascading relation as,

$$T_{12} = T_1 + \frac{T_2}{G_1} \tag{1}$$

where G_1 is the available power gain of the first network. It is common terminology to express mixer available power gain in terms of its reciprocal, L, the mixer conversion loss from signal to IF frequency. Identifying T_{12} as the receiver noise temperature T_R, T_1 as the mixer noise temperature T_M, and T_2 as the IF noise temperature T_{IF}, we obtain

$$T_R = T_M + LT_{IF} \tag{2}$$

The mixer noise temperature, T_M, as used here is a property only of the mixer and is not to be confused with T_R which is often described as the "mixer noise temperature" but depends upon T_{IF}.

The noise temperature, T_i, of the source resistance at the image 102
frequency is not included in T_M (but internal mixer image noise is to be included) and thus the overall system temperature, T_S, is given by

$$T_S = T_R + T_A + \frac{L}{L_i} T_i \tag{3}$$

where T_A is the source noise temperature at signal frequency and L_i is the conversion loss from image to IF frequency.

In the case of a signal which is broadband and contributes to both sidebands, the effective temperature is reduced by a factor of two, and also, for this case $T_i = T_A$ and $L_i = L$. Thus the double-sideband system temperature, T_S (DSB), is given by

$$T_S(\text{DSB}) = \frac{T_R}{2} + T_A \tag{4}$$

It is useful to define the double-sideband receiver temperature, T_R(DSB), as one-half of the single-sideband receiver temperature, T_R,[1]

$$T_R(\text{DSB}) \equiv \frac{T_R}{2} = \frac{T_M}{2} + \frac{L}{2} T_{IF} \tag{5}$$

---

[1]Note that the DSB noise figure $10 \cdot \log[T_R(\text{DSB})/T_o + 1]$ is not 3 dB less than the SSB noise figure. Also note that the noise generators used to measure T_R will contribute to both sidebands (unless the mixer is image-rejecting or has an image rejection filter) and T_R(DSB) is directly measured.

The mixer noise temperature, T_M, can be measured by measuring T_R, 103
T_{IF}, and L and using (2). However, it is common to describe the mixer noise performance in terms of a more directly measurable quantity, the mixer output noise temperature ratio, t. The quantity tT_o is by definition the mixer output noise temperature when the mixer source resistance for both signal and image is at temperature, $T_o = 290°$ K. It is related to T_M by

$$tT_o = \frac{T_o}{L} + \frac{T_o}{L_i} + \frac{T_M}{L} \qquad (6)$$

Output Noise Temp.	Converted Signal Noise	Converted Image Noise	Mixer Signal and Image Noise Referred to Output

Thus,

$$T_M = \left(Lt - 1 - L/L_i\right) T_o \qquad (7)$$

For a typical case $L = L_i = 5$, $t = 1.3$, $T_{IF} = 100$, we find $T_M = 1300$ and $LT_{IF} = 500$.

The quantity t is highly dependent upon conversion loss and is not a simple measure of the noise contribution of the mixer diode. In order to understand the sources of noise in a mixer and their dependence on physical temperature is useful to define a third temperature, T_D, the diode noise temperature[2]. This quantity is defined as the physical temperature of an attenuator having insertion loss L and L_i at signal and image temperatures and producing the same output noise temperature, tT_o, as the mixer:

$$tT_o = \frac{T_o}{L} + \frac{T_o}{L_i} + \left(1 - \frac{1}{L} - \frac{1}{L_i}\right) T_D \qquad (8)$$

The temperature ratio, t, can be measured and T_D can be found by inverting 104
(8) to give:

$$T_D = \frac{Lt - 1 - L/L_i}{L - 1 - L/L_i} \cdot T_o \qquad (9)$$

For the numerical examples given above, $T_D = 435°$. Alternatively, if T_M, L, and L_i have been measured, then T_D is given by

$$T_D = T_M / \left(L - 1 - L/L_i\right) \qquad (10)$$

Thus, for the case of $L = L_i$, the receiver and system temperatures can be expressed as

$$T_R = (L - 2)\, T_D + LT_{IF} \qquad (11)$$

$$T_S = T_A + T_i + (L - 2)\, T_D + LT_{IF} \qquad (12)$$

Total System Noise	Antenna Signal Noise	Antenna Image Noise	Mixer Noise	IF Noise

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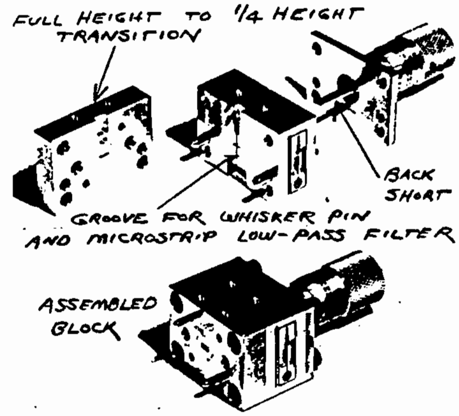
[2]For a non-linear resistive mixer T_D is equal to the DC-biased noise temperature T_{DC}, with a weighted average over the local oscillator current range. For an ideal Schottky diode exhibiting only shot-noise $T_D = T_{DC} =$ 1/2 physical temperature.

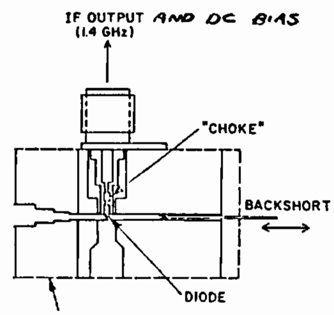
EXAMPLES OF MIXERS

MILLIMETER-WAVE (115 GHZ), SINGLE-DIODE WHISKER-CONTACTED MIXER AS DESIGNED BY KERR AND OTHERS



TYPICAL PERFORMANCE

f_{LO} GHZ	TEMP	CONV. LOSS L, DB	MIXER NOISE T_m
115	300	6.0	600 K
	15	5.5	150 K
230	300	8.5	1700 K
	15	7.5	500 K



AN EXCELLENT COLLECTION OF MIXER PAPERS IS GIVEN IN:

MICROWAVE AND MILLIMETER WAVE MIXERS, EDITED BY E.L. KOLLBERG, IEEE PRESS, 1984

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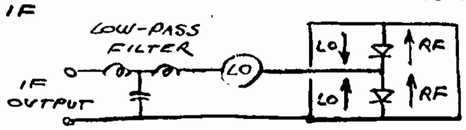
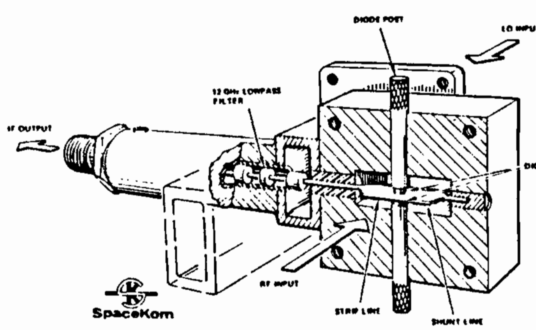
WAVEGUIDE BALANCED MIXER

BALANCED CIRCUIT ISOLATES LO AND RF PORTS AND ALSO CANCELS SECOND-HARMONIC RESPONSE.
IN CONFIGURATION AT RIGHT DIODES ARE IN SERIES FOR RF AND IN PARALLEL AT IF

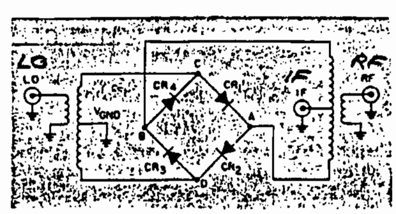




TYPICAL PERFORMANCE 7DB CONVERSION LOSS IN THE 12-50 GHZ RANGE

SEE CATALOG OF WATKINS-JOHNSON SANTA BARBARA, CA

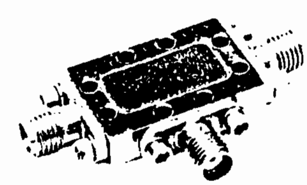
MICROWAVE COAXIAL DOUBLE-BALANCED MIXER



DOUBLE BALANCED MIXER ISOLATES RF FROM LO AND ALSO RF FROM IF (NO LOW-PASS FILTER NEEDED). WIDELY USED IN MICROWAVE RANGE WITH TYPICALLY 7DB CONVERSION LOSS.

A SINGLE MIXER WITH 2-18 GHZ RF AND LO RANGE AND 0.5-12 GHZ IF RANGE IS AVAILABLE (AVANTEK DBX18212)

SEE CATALOGS OF MINI-CIRCUITS, AVANTEK, AND VARI-L CORP.



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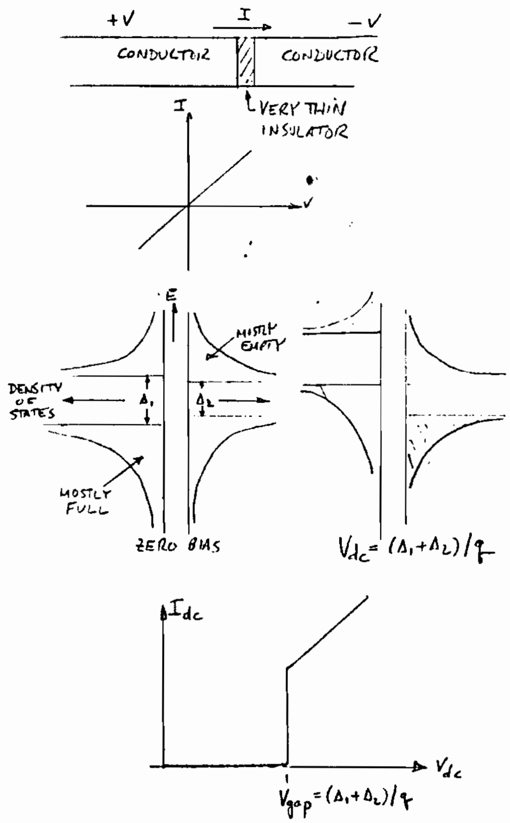
1. SIS JUNCTIONS

If two normal conductors are separated by a very thin insulating barrier, current can flow by quantum mechanical tunneling.

If the conductors are replaced by superconductors, the situation becomes more complicated because superconductors have an ENERGY GAP in their density of states functions. This gap has a width of a few milli electron volts (cf. 0.5 to 1 eV in semiconductors). At low temperatures (well below superconducting criticat temperature), the states below the gap are mostly full and those above are mostly empty. For small bias voltages, little current can flow. When the voltage raises the energy of electrons on one side to equal that of the empty states on the other side, a large current suddenly can flow. This results in an I-V curve with a very sharp non-linearity, useful for detection and mixing (Figure 1).

Practical junctions are constructed using microscopic lithography of thin films on insulating substrates (Figure 2). The insulating barriers are typically formed as metal oxides, and must be very thin-- 1 to 10 nm. Lateral dimensions of the junctions are 1 to 5 microns for those used in mixers. Notice that if it were not for the tunneling current these devices would be capacitors. This capacitance amounts to about 0.1 pf per micron of area, so the area must be kept small lest the resulting displacement current dominate the tunneling current at the operating frequency.



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Photon assisted tunneling:
If the junction is d.c. biased just below the gap voltage and an r.f. signal is applied, then the absorption of an r.f. photon can increase the energy of an electron enough to greatly increase its tunneling probability. For this to work, the photon should have an energy large compared to the sharpness of the I-V curve. For practical junctions, this is ΔV_g = 0.1 to 0.5 mV.

$$hf \gg q\Delta V_g \Rightarrow f \gg \frac{q}{h}\Delta V_g \approx 24 \text{ to } 120\ \text{GHz}$$

So photon assisted tunneling (p.a.t.) occurs at fairly high frequencies, typically in the millimeter wavelength range. At such frequencies, DETECTION OF INDIVIDUAL PHOTONS is possible with SIS junctions.

When p.a.t. is important, the a.c. response of the junction cannot be analyzed as if it were a non-linear resistor (as was done for Schottky diodes). The current does not respond instantaneously to the applied voltage. A detailed quantum mechanical analysis (see Tucker 1979, IEEE J. Quantum Elec., QE-15:1234; or Tucker and Feldman 1985, Rev. Mod. Phys. 57:1055) gives these formulas:

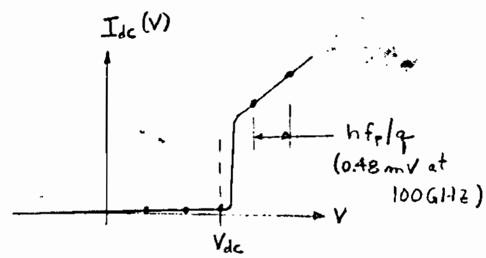
Applied voltage $V(t) = V_{dc} + V_{ac}(t)$

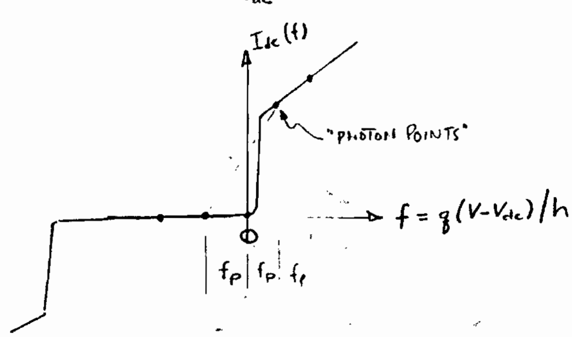
produces current

(1) $$I(t) = 2\,\mathrm{Re}\left\{ \int_{-\infty}^{t} I_{FT}(t-t')\, e^{j2\pi(q/h)\int_{t'}^{t} V_{ac}(\tau)\,d\tau}\, dt' \right\}$$

where

(2) $$I_{FT}(t) = \int_{-\infty}^{\infty} I_{dc}(hf/q - V_{dc})\, e^{j2\pi f t}\, df\,.$$





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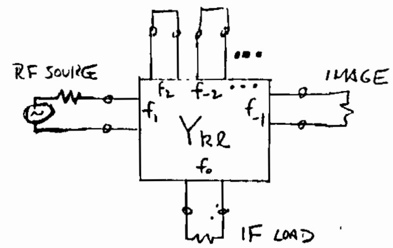
2. SIS MIXERS



Once we have an expression for the current response to an applied voltage, we can analyze the performance of a mixer in the same way as we did for the classical, resistive mixer: let the applied voltage consist of d.c. bias, pump (a periodic waveform, taken to be sinusoidal for simplicity), and small-signal $v(t)$ consisting of frequencies $f_k = k\, f_p + f_0$. We then calculate the components of small-signal current $i(t)$ at the same frequencies, obtaining the Y-matrix of a ficticious multiport network representing the pumped junction:

Classical -- $i(t) = g(t)\, v(t) = \sum_l Y_{kl}\, v_l\, e^{j2\pi f_l t}$

(3) $$g(t) = \left.\frac{dI_{dc}}{dV}\right|_{V=V_p(t)}$$

Quantum --

(4) $$i(t) = \underset{\text{from (1)}}{I(t)} - I_{dc} - I_p(t) = \sum_l Y_{kl}\, v_l\, e^{j2\pi f_l t}$$

(a) MIXER GAIN

One of the properties of SIS mixers that has made them interesting is that they can have conversion gain. This was predicted theoretically by Tucker (1980, Appl.Phys.Lett. 36:477) and later verified experimentally. It is easy to show that a resistive mixer cannot have gain (conversion loss < 1) in the classical limit, because the instantaneous power dissipated in the junction is

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(5) $$P(t) = i(t)\, v(t) = [g(t)\, v(t)]\, v(t) = g(t)\, [v(t)]^2,$$

which is positive for all t provided that $g(t)>0$. That is, there is always a net loss of signal power in the device; so, provided that the terminations at all frequencies are passive, the power delivered to the i.f. load can never exceed that available from the r.f. source.

In the quantum theory, there is no such restriction on the power dissipation, so gain might be possible. To see under what circumstances there can be gain and to determine its magnitude requires a complicated calculation (computer model). Some results are shown in Figure 3. Note that high gain is accompanied by high output impedance (low admittance), and that in fact the output admittance can go through zero and become negative, in which case the available gain is infinite.

(b) NOISE IN SIS MIXER RECEIVERS

Recall that the noise of a mixer receiver can be written

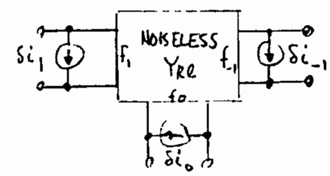
(6) $T_R = T_M + T_{IF}(Z_{out}) / G_M$

It might be hoped that the second term could be made negligible by having large mixer gain G_M and a low noise amplifier, but it should be noted that the amplifier noise depends on its source impedance, which is the mixer's output impedance. When the mixer has large gain, its output impedance may cause the amplifier noise to be large. In principle, a lossless coupling network can be introduced to achieve both high mixer gain and low amplifier noise, but this is limited by stability and saturation problems that we'll mention later.

The mixer's intrinsic noise T_M turns out to be very small is practical SIS mixers, and this is the principal reason for their attractiveness. The known physical mechanisms for this noise include shot noise in the junction, thermal noise in the junction, and thermal noise in the passive parts of the mixer circuit (waveguide losses, etc.). Much of the latter can be very small because the parts can be superconducting (note that the r.f. loss of superconductors is not zero like their d.c. loss, but is usually several orders of magnitude better than the best normal conductors at the same temperature--see Fig. 4). In the quantum theory, thermal and shot noise are unified; although (1) gives the expected value of the current induced by an a.c. voltage, the theory also gives the fluctuations about this value:

(7) $I_{TOTAL}(t) = I(t) + \delta I(t) = \langle \text{expected value (1)} \rangle + \langle \text{fluctuations} \rangle$

If we find the components of δI at all the mixing frequencies f_k, we can model the mixer noise as shown. When this is done in a careful computer calculation for practical junctions, it is often found that the noise temperature is close to the quantum limit. Experimental measurements of T_M have gotten as low as 3.8K at 36 GHz, which is within a factor of two of the quantum limit (Face et al. 1986, Appl.Phys.Lett. 48:1098)

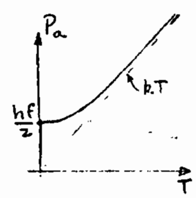


(c) THE QUANTUM LIMIT AND "QUANTUM NOISE"

The Heisenberg Uncertainty Principle imposes a limit on the noise temperature that can be achieved by any "high gain" linear device (mixer or amplifier); if it is "perfect" it still adds noise power of kT_{QL} per unit bandwidth, where

(8) $T_{QL} = hf/2k$.

This should not be understood as an additional source of noise, but rather as a statement that at least this much noise must be added by some (unspecified) mechanism. Calculations show that the thermal and shot noise will always account for this "quantum noise" in SIS mixers.



At high frequencies and/or low temperatures, care must be taken in accounting for thermal noise. The quantum theory shows that the available power spectral density from a resistor at temperature T is actually

(9) $P_a = (hf/2) \coth (hf/2kT)$; $\quad P_a \to kT$ as $hf/kT \to 0$

which is not quite zero at T=0. Nevertheless, we define noise temp. T_N to mean available noise power kT_N at all frequencies.

(d) PRACTICAL SIS MIXERS

Figures 7 and 6 show some details of construction of a practical SIS mixer for 100 GHz. A critical element in the design is the incorporation of a means of parallel-resonating the junction capacitance, otherwise it will tend to short out the signal. To achieve large bandwidth, the tuning elements should be close to the junctions, so it is advantageous to integrate them onto the same chip.

Series arrays of junctions are ususally used to increase the dynamic range. It turns out that the small signal approximations break down at fairly low power levels for SIS mixers, leading to saturation of the gain at inconveniently small input power. This saturation level is proportional to N^2 for N junctions in series, whereas the noise should be independent of the number of junctions.

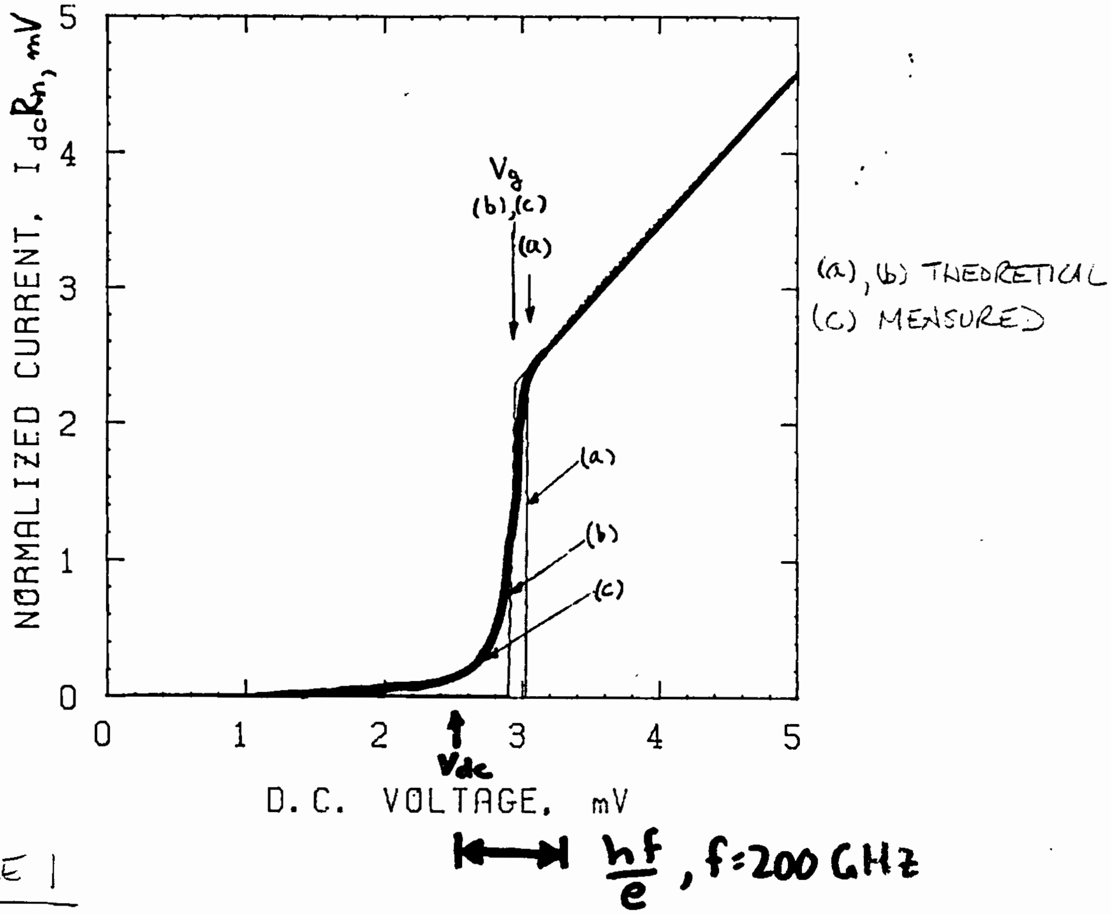


FIGURE 1

NIOBIUM-LEAD PROCESS

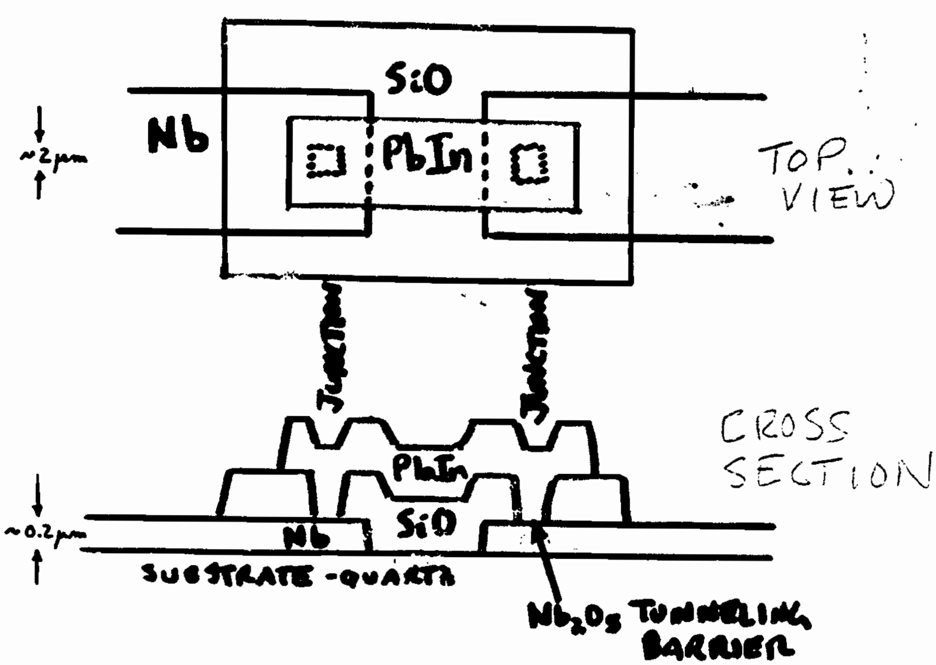


FIGURE 2

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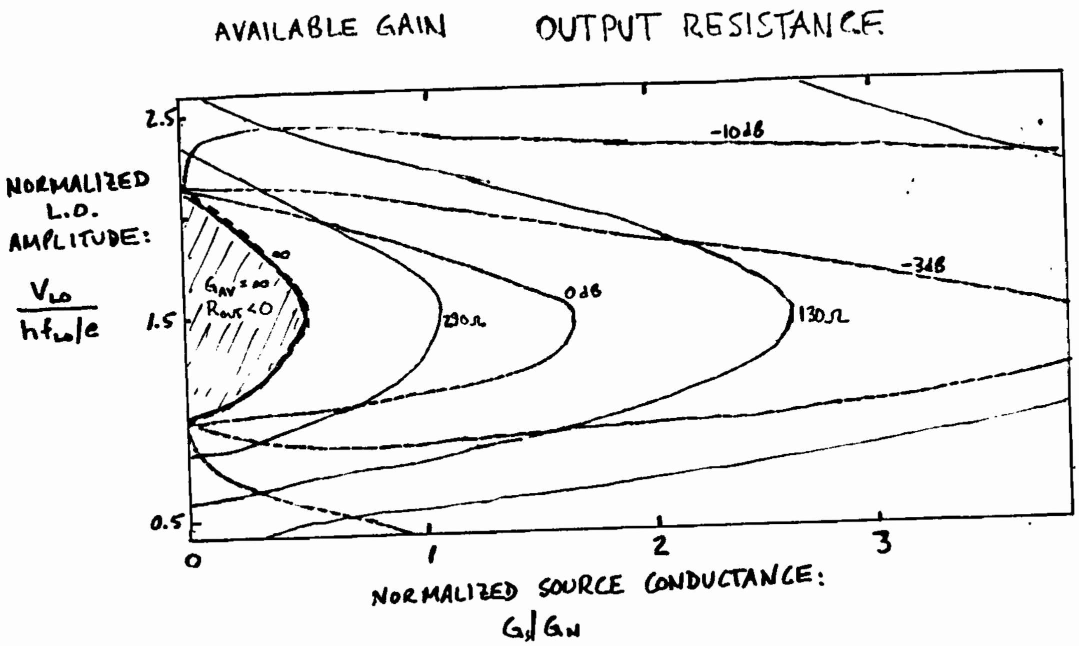

$B_s = 0$

$f_{LO} = 110$ GHz

FIG. 3

$f_{IF} = 1.5$ GHz $\qquad Y_i = Y_s$

$R_N = 78\,\Omega$ (U33-C28-D10)

12/85

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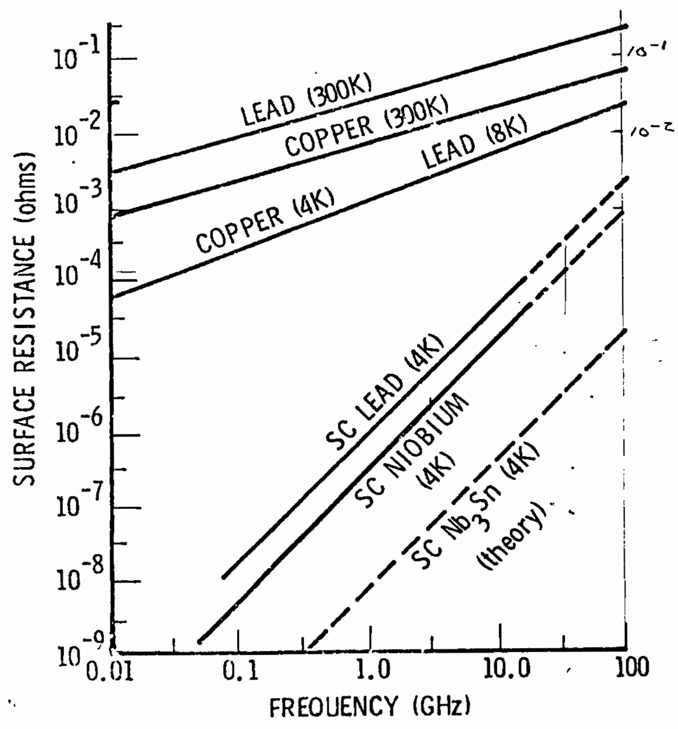

~~Figure 10.1~~

The microwave surface resistance of superconducting lead, niobium, and niobium-tin (Nb_3Sn) at 4 K. The solid lines represent experimental data, while the dashed lines are linear extrapolations. For comparison, the values for normal copper and lead are given at cryogenic and room temperature. [Hartwig and Passow, 1975].

FIG. 4

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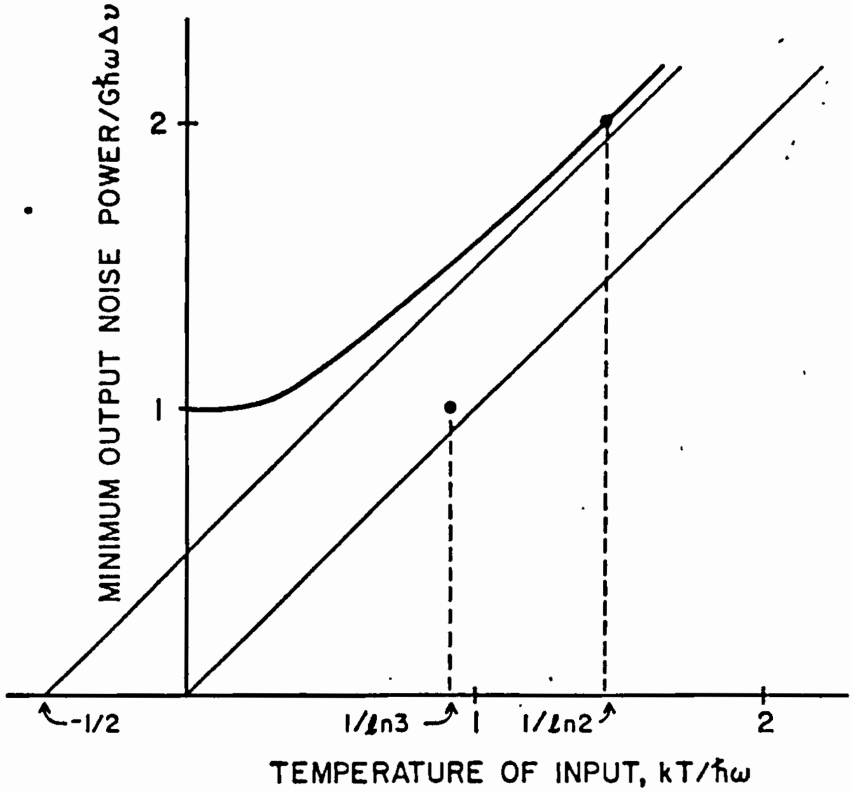

FIG. 5

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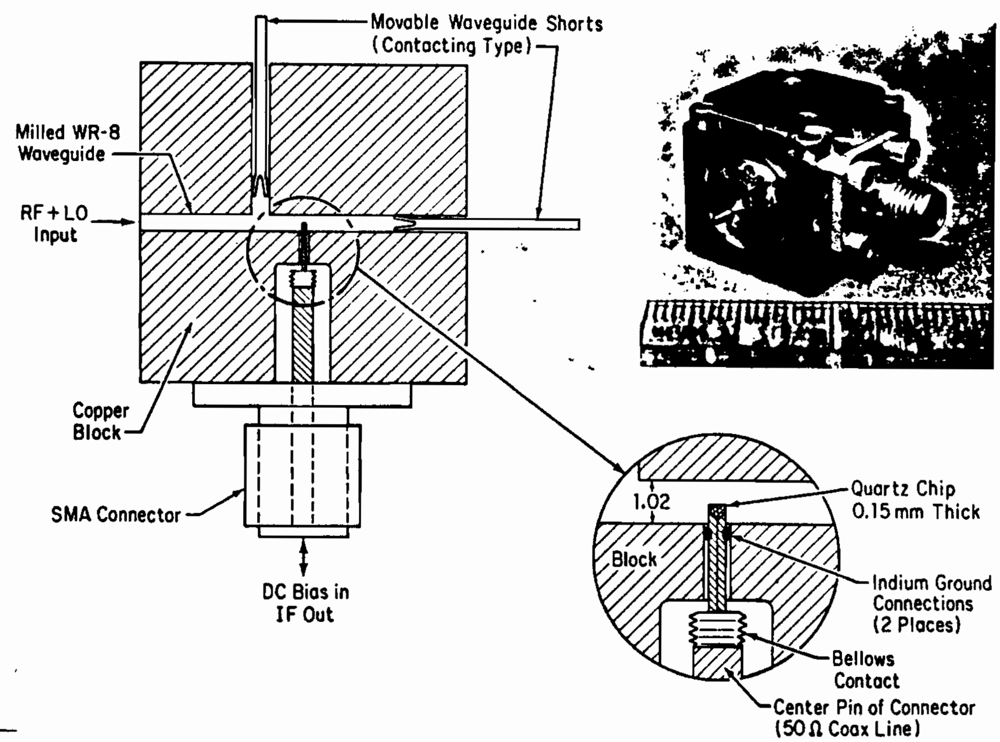


FIG. 6

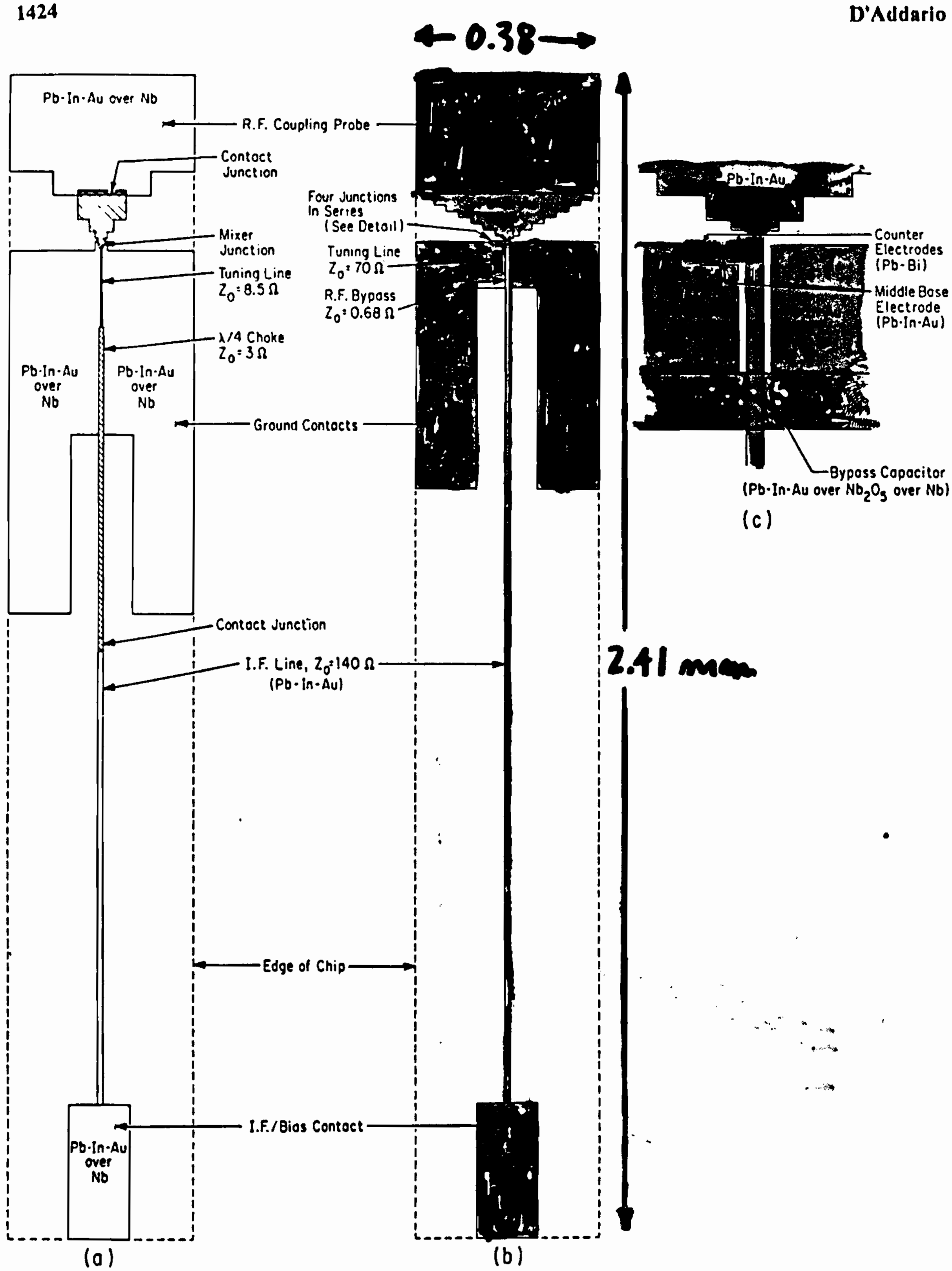
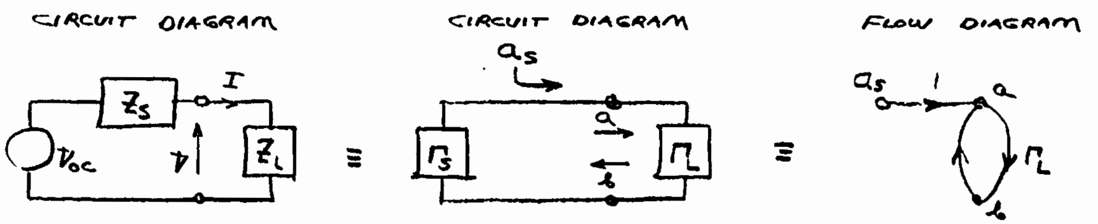

← 0.38 →
Pb-In-Au over Nb
R.F. Coupling Probe
Contact Junction
Four Junctions In Series (See Detail)
Mixer Junction
Tuning Line $Z_0 = 8.5\,\Omega$
Tuning Line $Z_0 = 70\,\Omega$
R.F. Bypass $Z_0 = 0.68\,\Omega$
λ/4 Choke $Z_0 = 3\,\Omega$
Pb-In-Au over Nb
Pb-In-Au over Nb
Ground Contacts
Contact Junction
I.F. Line, $Z_0 = 140\,\Omega$ (Pb-In-Au)
2.41 mm
Edge of Chip
I.F./Bias Contact
Pb-In-Au over Nb
(a)
(b)
Pb-In-Au
Counter Electrodes (Pb-Bi)
Middle Base Electrode (Pb-In-Au)
Bypass Capacitor (Pb-In-Au over Nb_2O_5 over Nb)
(c)

FIGURE 7

POWER GAIN DEFINITIONS AND STABILITY

LETS FIRST REVIEW THE CONCEPTS OF AVAILABLE POWER P_{AVS} OF A SOURCE AND DELIVERED POWER P_L TO A LOAD IN TERMS OF VOLTAGE AND IMPEDENCE AND WAVES AND REFLECTION COEFFICIENT. SINCE WE ARE DISCUSSING POWER LETS USE RMS VALUES FOR VOLTAGES AND WAVES (POWER IN A WAVE IS $|a|^2$ RATHER THAN $|a|^2/2$). CONSIDER THE FOLLOWING REPRESENTATIONS OF A SOURCE AND LOAD:



WE RELATE THESE DIAGRAMS BY $\Gamma = (Z-Z_0)/(Z+Z_0)$ AND FINDING a_S IN TERMS OF V_{oc}

THIS EQ. DEFINES a_S IN ABOVE MIDDLE DIAGRAM $\longrightarrow$ $a = a_S + b\Gamma_S$

$b = a\Gamma_L$

THEN $a = a_S/(1-\Gamma_S\Gamma_L)$

$V_{oc} = (a+b)\sqrt{Z_0}$ @ $\Gamma_L = 1$

$$V_{oc} = \frac{2a_S\sqrt{Z_0}}{1-\Gamma_S}$$

OR

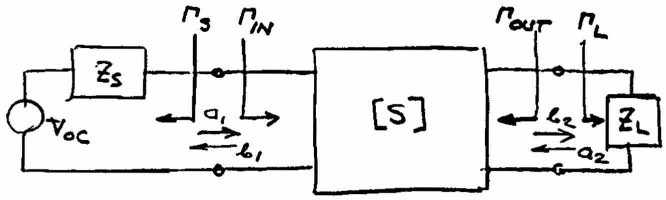
$$a_S = \frac{V_{oc}}{2\sqrt{Z_0}}(1-\Gamma_S)$$

WE CAN THEN MAKE THE FOLLOWING TABLE:

SYMBOL	DEFINITION	IN TERMS OF V_{oc}, Z_S, AND Z_L	IN TERMS OF a_S, Γ_S, AND Γ_L
P_L	POWER DELIVERED (AND ABSORBED) BY LOAD	$\dfrac{\lvert V_{oc}\rvert^2 R_L}{\lvert Z_S+Z_L\rvert^2}$	$\dfrac{\lvert a_S\rvert^2(1-\lvert\Gamma_L\rvert^2)}{\lvert 1-\Gamma_S\Gamma_L\rvert^2}$
P_L	AS ABOVE IN TERMS OF P_{AVS}	$P_{AVS}\cdot\dfrac{4R_S R_L}{\lvert Z_S+Z_L\rvert^2}$	$P_{AVS}\cdot\dfrac{(1-\lvert\Gamma_S\rvert^2)(1-\lvert\Gamma_L\rvert^2)}{\lvert 1-\Gamma_S\Gamma_L\rvert^2}$
P_{AVS}	POWER AVAILABLE FROM SOURCE	$\lvert V_{oc}\rvert^2/(4R_S)$	$\lvert a_S\rvert^2/(1-\lvert\Gamma_S\rvert^2)$
P_o	POWER DELIVERED TO A Z_0 LOAD	$\dfrac{\lvert V_{oc}\rvert^2 Z_0}{\lvert Z_S+Z_0\rvert^2}$	$\lvert a_S\rvert^2 = P_{AVS}(1-\lvert\Gamma_S\rvert^2)$

(V_{oc} AND a_S ARE RMS VALUES)

APPLICATION TO TWO-PORT NETWORKS



WE WILL FIRST FIND THE <u>TRANSDUCER GAIN</u> $G_T \equiv P_L/P_{AVS}$ BY THE FOLLOWING STEPS:

1) FIND $|a_1|^2$ IN TERMS OF P_{AVS} FROM PROCEDING PAGE

$$|a_1|^2 = |a_S|^2/|1-\Gamma_S\Gamma_{IN}|^2 = P_{AVS}(1-|\Gamma_S|^2)/|1-\Gamma_S\Gamma_{IN}|^2$$

2) FIND $|b_2|^2$ IN TERMS OF $|a_1|^2$ BY USING

$$b_2 = S_{21}a_1 + S_{22}a_2 \qquad a_2 = b_2\Gamma_L$$

$$|b_2|^2 = |S_{21}|^2|a_1|^2/|1-S_{22}\Gamma_L|^2$$

3) FIND P_L IN TERMS OF $|b_2|^2$

$$P_L = |b_2|^2(1-|\Gamma_L|^2)$$

THESE STEPS ARE THEN PUT TOGETHER TO GIVE THE FOLLOWING EQUATIONS, ALL GIVING TRANSDUCER GAIN:

$$G_T \equiv \frac{P_L}{P_{AVS}} = \underbrace{\frac{(1-|\Gamma_S|^2)}{|1-\Gamma_{IN}\Gamma_S|^2}}_{\text{SOMETIMES TERMED } S_{11}'} \cdot |S_{21}|^2 \cdot \underbrace{\frac{(1-|\Gamma_L|^2)}{|1-S_{22}\Gamma_L|^2}}_{\text{NOTE } S_{22}\text{, NOT } \Gamma_{OUT} \text{ OR } S_{22}'}$$

IN TERMS OF Γ_{IN}, AND S

OTHER FORMS ARE:

$$G_T \equiv \frac{P_L}{P_{AVS}} = \frac{(1-|\Gamma_S|^2)}{|1-S_{11}\Gamma_S|^2} \cdot |S_{21}|^2 \cdot \frac{(1-|\Gamma_L|^2)}{|1-\Gamma_{OUT}\Gamma_L|^2}$$

IN TERMS OF Γ_{OUT}, AND S

$$G_T \equiv \frac{P_L}{P_{AVS}} = \frac{(1-|\Gamma_S|^2) \cdot |S_{21}|^2 \cdot (1-|\Gamma_L|^2)}{|(1-S_{11}\Gamma_S)(1-S_{22}\Gamma_L) - S_{21}S_{12}\Gamma_L\Gamma_S|^2}$$

IN TERMS OF ONLY S

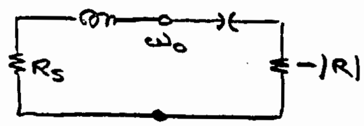
WHERE

$$\Gamma_{IN} \equiv S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L} \qquad \Gamma_{OUT} \equiv S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S}$$

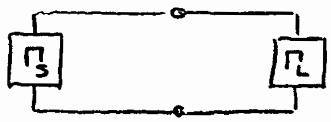
SINCE Γ_{IN} IS A FUNCTION OF Γ_L AND Γ_{OUT} IS A FUNCTION OF Γ_S IT IS NOT A SIMPLE MATTER TO MATCH INPUT AND OUTPUT.
WE SET $\Gamma_L = \Gamma_{OUT}^*$ AND $\Gamma_S = \Gamma_{IN}^*$ TO FORM EQUATIONS WHICH HAVE A SOLUTION WHEN A STABILITY FACTOR $K > 1$ TO BE DEFINED NEXT.
THESE SOLUTIONS FOR MATCHED LOAD AND SOURCE, Γ_{ML} AND Γ_{MS}, ARE CALLED <u>SIMULTANEOUS BILATERAL MATCH</u> AND ARE GIVEN IN THE AVANTEK PRIMER II p.5 OR GONZALEZ p. 113 OR NOTES p 128

STABILITY

AN ACTIVE CIRCUIT MAY OSCILLATE BY PRODUCING NEGATIVE RESISTANCE HAVING MAGNITUDE > EXTERNAL POSITIVE RESISTANCE. FOR EXAMPLE:



THIS LOOP WILL OSCILLATE AT ω_0 IF $R_S \leq |R|$. THE $-|R|$ CAN ARISE AT THE INPUT OR OUTPUT OF A TWO-PORT AND BE A FUNCTION OF THE TERMINATION OF THE OTHER PORT.



STATED IN TERMS OF REFLECTION COEFFICIENTS, THIS LOOP WILL OSCILLATE IF $|\Gamma_S||\Gamma_L| \geq 1$. IF IT IS TO BE STABLE FOR ANY $|\Gamma_S| \leq 1$ THEN $\Gamma_L < 1$.

A TWO-PORT NETWORK IS STABLE IF $|\Gamma_{IN}|$ AND $|\Gamma_{OUT}|$ ARE ≤ 1 FOR ANY PASSIVE LOAD AT THE OPPOSITE PORT. A NECESSARY AND SUFFICIENT CONDITIONS FOR STABILITY ARE:

$$\text{STABILITY FACTOR} \equiv K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + D^2}{2|S_{12}S_{21}|} > 1 \quad \text{AND} \quad D < 1$$

WHERE $D = |S_{11}S_{22} - S_{12}S_{21}|$

INVARIANT TO LOSSLESS TRANSFORMATIONS, INTERCHANGE OF PORTS 1 AND 2, OR USING Z OR Y IN PLACE OF S!

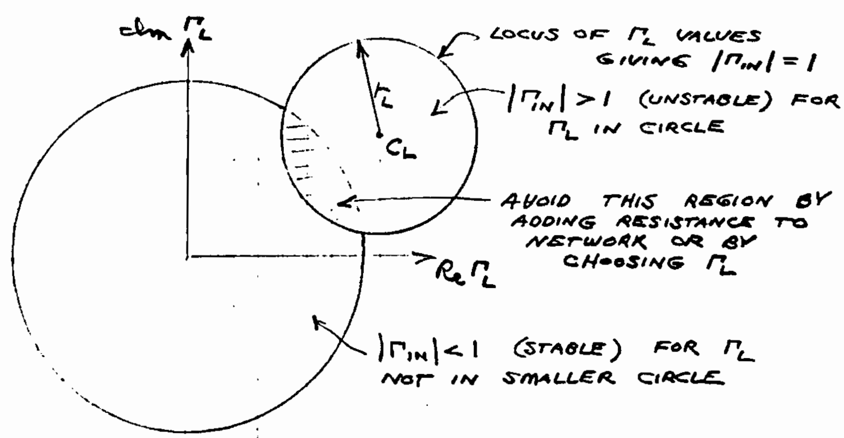
REFERENCES:

ROLLET, "STABILITY AND POWER-GAIN INVARIANTS...", IEEE TRANS. CT-9, 3/62
GONZALEZ, MICROWAVE TRANSISTOR AMP..., PRENTICE HALL, 1984
AVANTEK HF TRANSISTOR PRIMER - PART II - 408-970-2583
HP APPLICATION NOTE 154, "S PARAMETER DESIGN"

STABILITY CIRCLES

IF A NETWORK OR DEVICE HAS $K < 1$ (ALMOST ALL TRANSISTORS DO AT SOME FREQUENCIES) IT IS STILL USABLE BY CHOOSING LOAD Γ_L AND SOURCE Γ_S WHICH DO NOT PRODUCE $|\Gamma_{IN}| > 1$ OR $|\Gamma_{OUT}| > 1$. THIS CAN BE UNDERSTOOD BY CONSIDERING THE LOCUS OF LOAD Γ_L WHICH PRODUCES $|\Gamma_{IN}| = 1$. THIS PRODUCES A CIRCLE IN THE Γ_L PLANE AS SHOWN BELOW:

(EQUATIONS FOR Γ_L AND C_L ARE GIVEN ON p. 128)



A SIMILAR CONSTRUCTION CAN BE MADE IN THE Γ_S PLANE. THE RADII AND CENTERS OF THESE CIRCLES ARE FUNCTIONS OF S PARAMETERS AND ARE GIVEN IN HP APPLICATION NOTE 154 p.13, AVANTEK PRIMER II, p.7 OR GONZALEZ p.96

POWER GAIN SUMMARY TABLE

SYMBOL	NAME	FUNCTION OF OR CONDITION ON Γ_S	Γ_L	COMMENT
G_T	TRANSDUCER GAIN	YES	YES	SEE EQUATIONS ON P. 124
G_U	UNILATERAL GAIN	YES	YES	G_T WITH $S_{12}=0$, $\Gamma_{IN} = S_{11}$, $\Gamma_{OUT} = S_{22}$
G_{Umax}	MAX UNILATERAL GAIN	S_{11}^*	S_{22}^*	G_U AT $\Gamma_S = S_{11}^*$, $\Gamma_L = S_{22}^*$
G_O	INSERTION GAIN IN Z_0 SYSTEM	0	0	$G_O = \lvert S_{21}\rvert^2$
G_{AV}	AVAILABLE GAIN	YES	Γ_{OUT}^*	$\equiv P_{AV}(\text{NETWORK})/P_{AVS} = G_T$ WITH $\Gamma_L = \Gamma_{OUT}^*$ USED IN NOISE TEMP CASCADE FORMULA. CONSTANT FOR Γ_S ON LOCUS OF CIRCLE
G_{MAG}	MAX AVAILABLE GAIN $G_{MAG} = \left\lvert\frac{S_{21}}{S_{12}}\right\rvert (K-\sqrt{K^2-1})$ IF $K>1$	Γ_{IN}^*	Γ_{OUT}^*	FUNDAMENTAL PROPERTY OF TWO-PORT, FUNCTION OF S PARAMETERS ONLY INVARIANT TO Γ_S OR Γ_L AND HENCE INVARIANT TO LOSSLESS INPUT OR OUTPUT NETWORKS. NOT INVARIANT TO FEEDBACK
U	MASON'S GAIN = G_{MAG} OF TWO-PORT WITH LOSSLESS EMBEDDING TO MAKE $S_{12}=0$. $U=0$ FOR RECIPROCAL NETWORK	NO	NO	$\lvert (S_{21}/S_{12})-1\rvert^2/[2K\lvert S_{21}/S_{12}\rvert - 2\,\mathrm{Re}(S_{21}/S_{12})]$ REF: S. MASON, "POWER GAIN IN FEEDBACK AMPLIFIERS," IRE TRANS CT-1, JUNE, 1954, PP. 20-25. IT IS THE MAXIMUM AVAILABLE GAIN OF THE UNILATERALIZED NETWORK AND IS INVARIANT.
G_{NA}	NOISE ASSOCIATED GAIN	Γ_{OPT}	NO	AVAILABLE GAIN AT SOURCE IMPEDENCE CHOSEN FOR MINIMUM NOISE

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CLASSIFICATION OF NETWORKS IN TERMS OF K, S_{12}, AND G_{MAG}

K	S_{12}	CLASS	G_{MAG}
∞	0	UNILATERAL	$\lvert S_{21}\rvert^2/[(1-\lvert S_{11}\rvert^2)(1-\lvert S_{22}\rvert^2)]$
$1<K<\infty$	SMALL	STABLE ACTIVE	$\left\lvert\frac{S_{21}}{S_{12}}\right\rvert(K-\sqrt{K^2-1}) \rightarrow \left\lvert\frac{S_{21}}{S_{12}}\right\rvert\cdot\frac{1}{2K}$ FOR $K \gg 1$
$1<K<\infty$	$S_{12}=S_{21}$	LOSSY RECIPROCAL	$G_{MAG} < 1$
1	MODERATE	ACTIVE	$\left\lvert\frac{S_{21}}{S_{12}}\right\rvert$
1	$S_{12}=S_{21}$	LOSSLESS, RECIPROCAL	$G_{MAG} = 1$
<1	LARGER	POTENTIALLY UNSTABLE	$G_{MAG}=\infty$, CAN BE MADE STABLE BY ADDITION OF LOSS AT INPUT OR OUTPUT

STABILITY CIRCLE EQUATIONS

Γ_L values for $\lvert\Gamma_{IN}\rvert = 1$ (Output Stability Circle):

$$r_L = \left\lvert \frac{S_{12}S_{21}}{\lvert S_{22}\rvert^2 - \lvert\Delta\rvert^2}\right\rvert \quad \text{(radius)}$$

$$C_L = \frac{(S_{22}-\Delta S_{11}^*)^*}{\lvert S_{22}\rvert^2 - \lvert\Delta\rvert^2} \quad \text{(center)}$$

Γ_s values for $\lvert\Gamma_{OUT}\rvert = 1$ (Input Stability Circle):

$$r_s = \left\lvert \frac{S_{12}S_{21}}{\lvert S_{11}\rvert^2 - \lvert\Delta\rvert^2}\right\rvert \quad \text{(radius)}$$

$$C_s = \frac{(S_{11}-\Delta S_{22}^*)^*}{\lvert S_{11}\rvert^2 - \lvert\Delta\rvert^2} \quad \text{(center)}$$

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

SIMULTANEOUS BILATERAL MATCH, Γ_{MS} AND Γ_{ML}

$$\Gamma_{MS} = \frac{B_1 \pm \sqrt{B_1^2 - 4\lvert C_1\rvert^2}}{2C_1}$$

USE + SIGN WHEN $B_1<0$

$$\Gamma_{ML} = \frac{B_2 \pm \sqrt{B_2^2 - 4\lvert C_2\rvert^2}}{2C_2}$$

USE + SIGN WHEN $B_2<0$

$$B_1 = 1 + \lvert S_{11}\rvert^2 - \lvert S_{22}\rvert^2 - \lvert\Delta\rvert^2$$

$$B_2 = 1 + \lvert S_{22}\rvert^2 - \lvert S_{11}\rvert^2 - \lvert\Delta\rvert^2$$

$$C_1 = S_{11} - \Delta S_{22}^*$$

$$C_2 = S_{22} - \Delta S_{11}^*$$

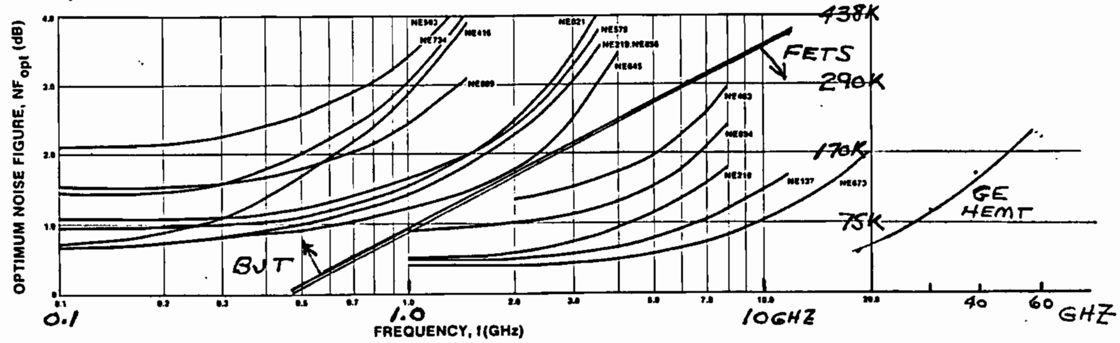
$$\Delta = \det[S] = S_{11}S_{22} - S_{12}S_{21}$$

VALID FOR $K>1$

MICROWAVE TRANSISTORS

ABBREV	NAME	MATERIAL	TYP f_{MAX}	TYP COST	FREQUENCY RANGE
BJT	BIPOLAR JUNCTION TRANSISTOR	SILICON	10 GHZ	\$2	< 4 GHZ
FET (GASFET)	FIELD - EFFECT TRANSISTOR	GaAs	40 GHZ	\$30	2 TO 20 GHZ
HEMT (MODFET)	HIGH - ELECTRON MOBILITY TRANSISTOR	GaAs/ AlGaAs	80 GHZ	\$100	10 TO 60 GHZ

Typical Optimum Noise Figure vs. Frequency for Bipolar Transistors and FETs NEC 1986

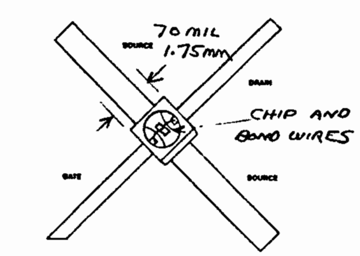


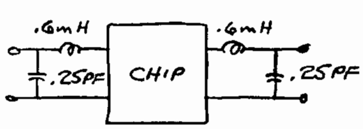
POWER OUTPUT

BJT'S 100W @ 400 MHZ
10W @ 2 GHZ

FET'S 10W @ 2GHZ
1W @ 10 GHZ

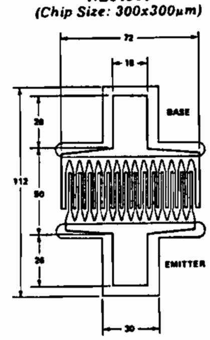
TRANSISTOR PACKAGE

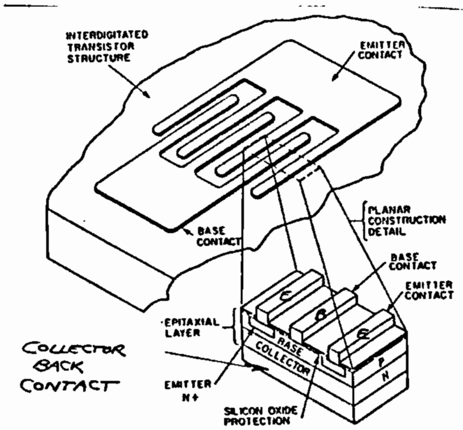




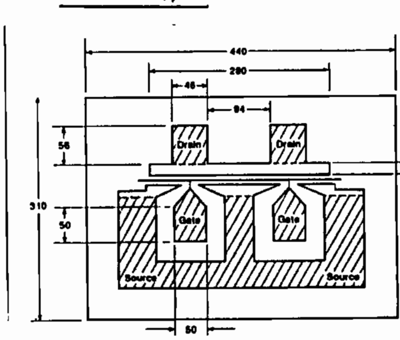
BJT CHIP

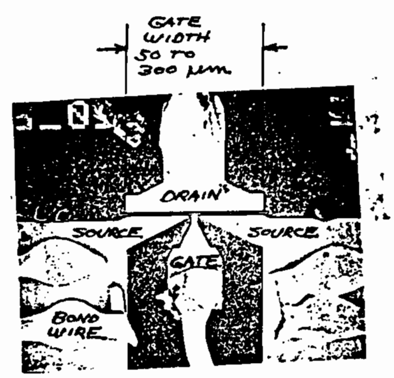
NE64500
(Chip Size: 300x300μm)

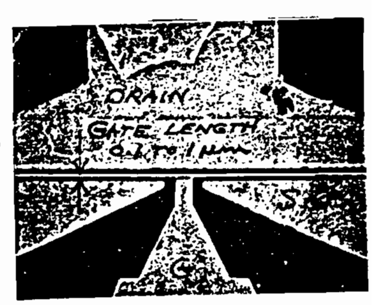




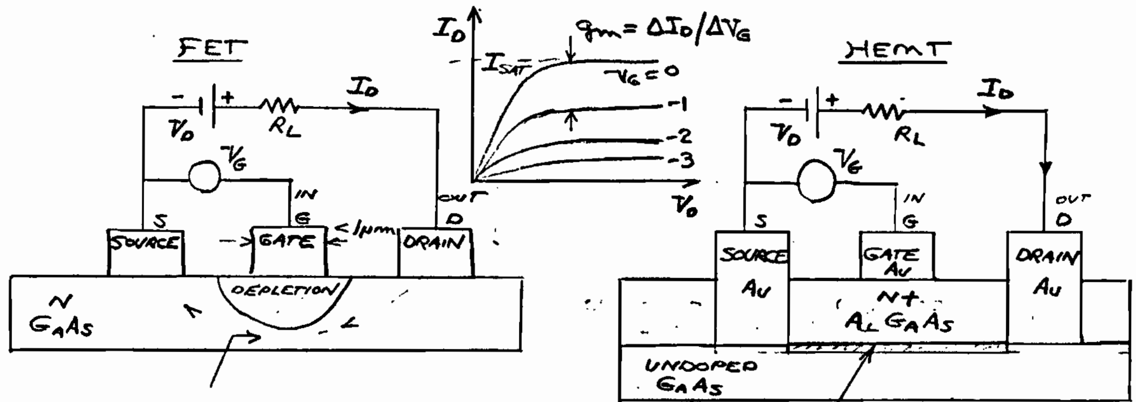
FET CHIP







PRINCIPLES OF OPERATION OF FET AND HEMT DEVICES



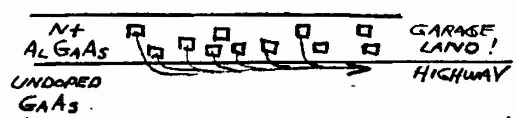
DRAIN TO SOURCE CONDUCTION THRU CHANNEL WITH WIDTH DEPENDENT ON V_G

DRAIN TO SOURCE CONDUCTION THRU TWO-DIMENSIONAL ELECTRON GAS IN GaAs WITH ELECTRONS DRAWN FROM AlGaAs

HIGHWAY MODEL OF FET

GARAGES (DONOR SITES) IN MIDDLE OF ROAD LIMIT CARRIER VELOCITY
FLOW CONTROLLED BY HIGHWAY WIDTH

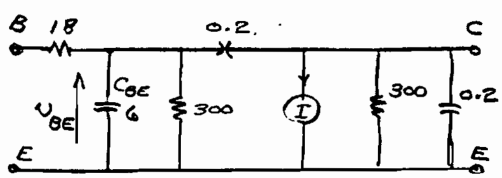
HIGHWAY MODEL OF HEMT



GARAGES ON SIDE OF ROAD!
FLOW CONTROLLED BY NUMBER OF CARS

BJT AND FET EQUIVALENT CIRCUITS AND S PARAMETERS

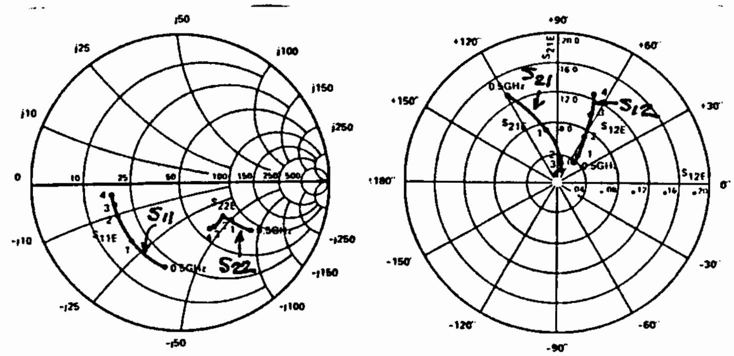
NE64500 BJT CHIP



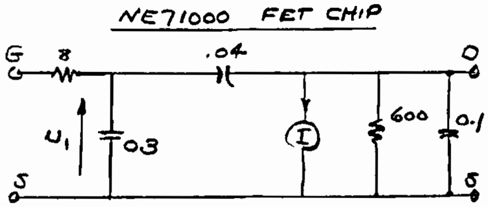
$I = g_m v_{BE}$

$g_m = 2\pi C_{BE} f_T$

$f_{max} = 8.5$ GHZ ~ 3 S



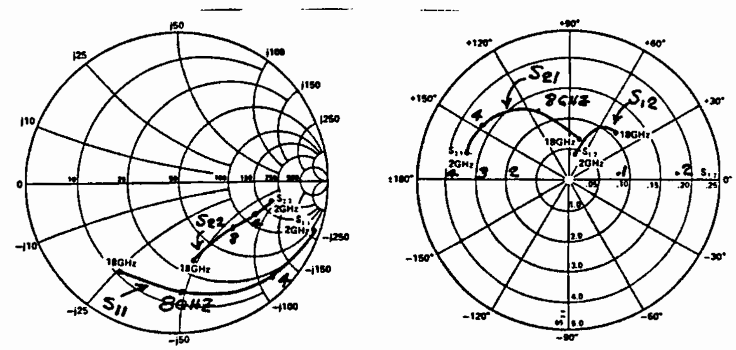
NE71000 FET CHIP



$I = g_m v_1$

$g_m = .05$ S

$f_{max} \sim 50$ GHZ



TRANSISTOR AMPLIFIERS   REF: MICROWAVE TRANSISTOR AMPLIFIERS, G. GONZALEZ, PRENTICE HALL, 1984

- AMPLIFIERS ARE USUALLY MATCHED ($S_{11} = S_{22} = 0$) SO THEY CAN BE CASCADED WITH OTHER COMPONENTS, EVEN WITH INTERCONNECTING TRANSMISSION LINES WITHOUT REFLECTIONS. IF REFLECTIONS ARE PRESENT THE RESULTING FREQUENCY RESPONSE CONTAINS RIPPLES AND IS DEPENDENT ON THE LENGTH OF INTERCONNECTING T-LINE.

- IF THE TRANSISTOR HAS $K > 1$ THEN INPUT AND OUTPUT MATCHING NETWORKS WHICH TRANSFORM Z_0 TO Γ_{mS} AND Γ_{mL} (NOTES (128)) CAN BE DESIGNED. CARE MUST BE TAKEN THAT THE TRANSISTOR IS NOT TERMINATED IN IMPEDENCES THAT CAUSE OSCILLATION AT ANY FREQUENCY WHERE $K < 1$.

- THE RESULTING MATCHED AMPLIFIER IS NOT UNILATERAL ($S_{12} \neq 0$) AND Γ_{IN} IS A FUNCTION OF Γ_L BUT THIS MAY BE TOLERABLE AND CAN BE IMPROVED BY ADDITION OF AN ISOLATOR AT INPUT OR OUTPUT. FEEDBACK CAN MAKE THE AMPLIFIER UNILATERAL BUT IT MAY BE UNSTABLE AT OTHER FREQUENCIES.

LOW NOISE AMPLIFIERS   REF: LOW NOISE MICROWAVE TRANSISTORS AND AMPLIFIERS, EDITED BY H. FUKUI, IEEE PRESS, 1981

TRANSISTOR NOISE PARAMETERS MUST BE KNOWN. IF THEY ARE KNOWN FOR A FET CHIP AT ONE FREQUENCY THEY CAN BE DETERMINED FOR ANOTHER FREQUENCY BY USING THE THEORETICAL FREQUENCY DEPENDENCE OF FET NOISE PARAMETERS (REF: PUCEL, STATZ, AND HAUS, ADVANCES IN ELECTRONICS AND ELECTRON PHYSICS, VOL 38 ACADEMIC, 1975)

$$T_{MIN} = A \cdot f \qquad N = B \cdot f \qquad R_{OPT} = C/f \qquad X_{OPT} = D/f$$

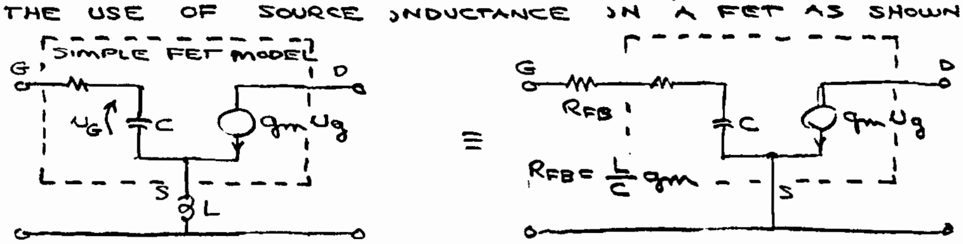
THE AMPLIFIER INPUT NETWORK MUST BE DESIGNED TO TRANSFORM Z_0 TO Z_{OPT} AND OUTPUT NETWORK MUST TRANSFORM Γ_{OUT} TO Z_0

NOISE OPTIMIZATION (SOMETIMES CALL NOISE MATCH) OF THE INPUT USUALLY DOES NOT PROVIDE POWER MATCH; THAT IS $\Gamma_{OPT} \neq \Gamma_{IN}^*$

SIMULTANEOUS NOISE AND POWER MATCH IS POSSIBLE BY:

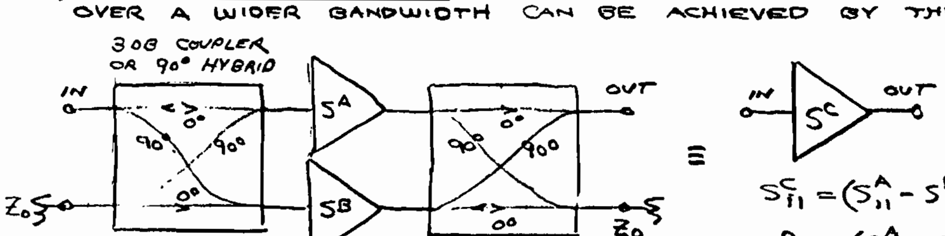
1) INPUT ISOLATOR – THE LOSS OF THE ISOLATOR (TYPICALLY 0.5 DB) ADDS TO THE INPUT NOISE BUT MAY BE TOLERABLE

2) LOSSLESS FEEDBACK – THIS CAN BE USED TO MODIFY Γ_{IN} WHILE HAVING A SMALLER EFFECT ON Γ_{OPT}. AN EXAMPLE IS THE USE OF SOURCE INDUCTANCE IN A FET AS SHOWN BELOW:



THE INDUCTANCE L INCREASES THE INPUT RESISTANCE BY R_{FB} (FOR $L = 0.4$ nH, $C = 0.5$ PF, AND $G_m = .05$ $R_{FB} = 40$) WHILE THE EFFECT ON Z_{OPT} IS TO REDUCE IT BY $j\omega L$ ($j4$ @ 1.5 GHZ).

3) BALANCED AMPLIFIERS – THE EFFECT OF AN ISOLATOR BUT OVER A WIDER BANDWIDTH CAN BE ACHIEVED BY THE FOLLOWING



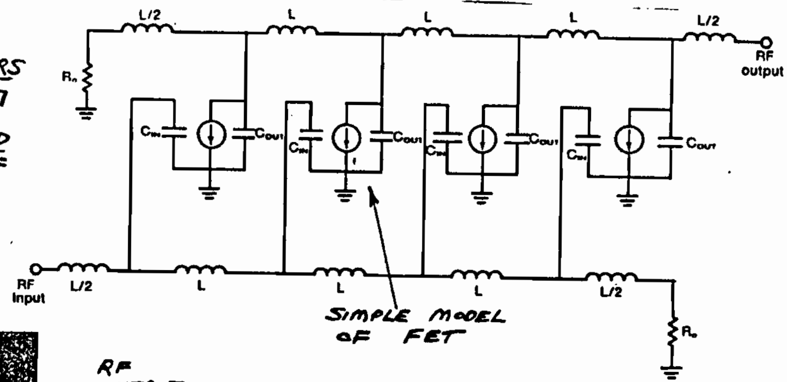
$$S_{21}^C = (S_{21}^A + S_{21}^B)/2$$
$$S_{12}^C = (S_{12}^A + S_{12}^B)/2$$
$$S_{11}^C = (S_{11}^A - S_{11}^B)/2$$
$$S_{22}^C = (S_{22}^A - S_{22}^B)/2$$

DISTRIBUTED AMPLIFIERS

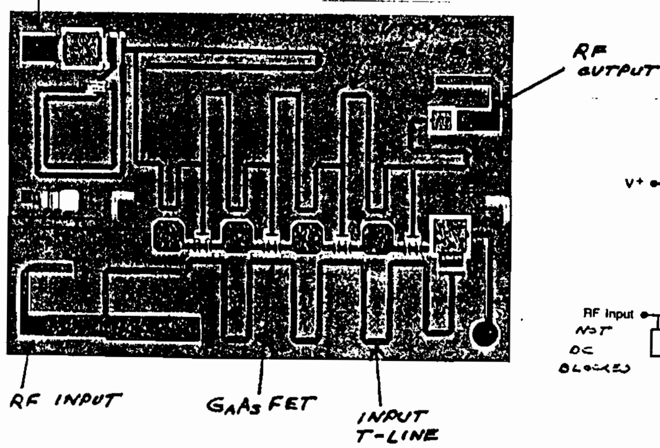
VERY WIDEBAND (2 TO 26 GHZ) FET AMPLIFIERS CAN BE REALIZED BY INCORPORATING FETS INTO A TRANSMISSION LINE AS SHOWN BELOW:

REF: NICLAS ET AL, ON THEORY AND PERFORMANCE OF SOLID-STATE MICROWAVE DISTRIBUTED AMPLIFIERS IEEE MTT-31, JUNE 1983, pp. 447

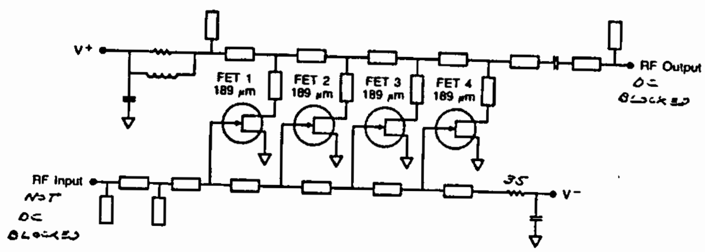
KENNAN AND OSBRINK, "DISTRIBUTED AMPLIFIERS ···", MICROWAVES AND RF NOVEMBER, 1984 pp. 119-127



TEXAS INSTRUMENTS TGA 8300
.064" x .093" CHIP ~ $100
+6V, 100 mA

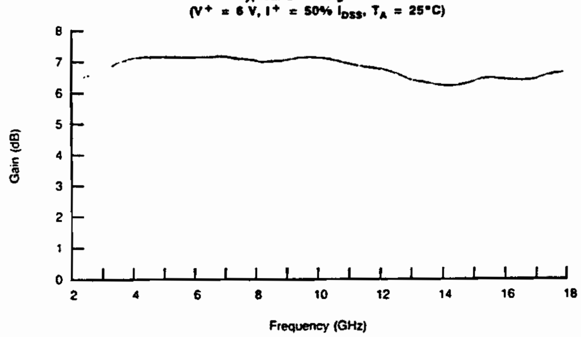


Circuit Topology

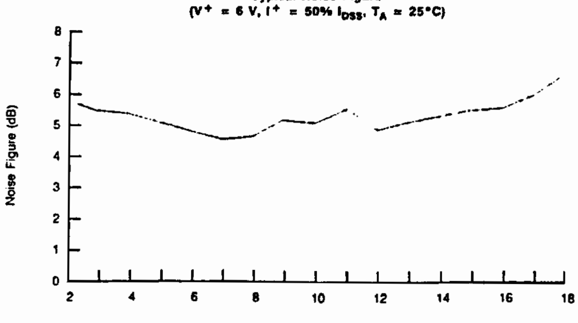


PERFORMANCE OF TGA 8300 MONOLITHIC DISTRIBUTED AMPLIFIER

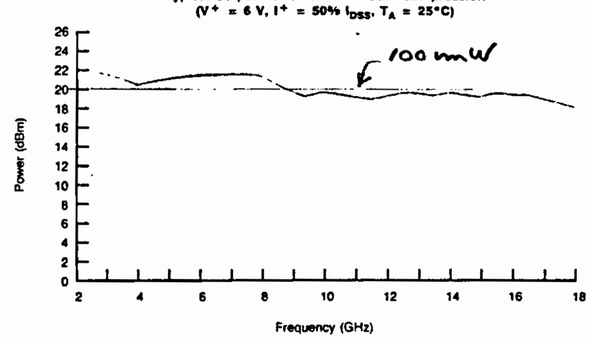
Typical Small-Signal Gain
($V^+ = 6$ V, $I^+ = 50\%$ I_{DSS}, $T_A = 25°C$)



Typical Noise Figure
($V^+ = 6$ V, $I^+ = 50\%$ I_{DSS}, $T_A = 25°C$)



Typical Output Power at 1-dB Gain Compression
($V^+ = 6$ V, $I^+ = 50\%$ I_{DSS}, $T_A = 25°C$)



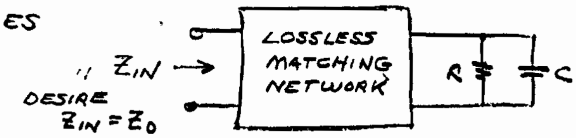
Typical S-Parameters
($V^+ = 6$ V, $I^+ = 50\%$ I_{DSS}, $T_A = 25°C$)

Frequency (GHz)	S_{11}		S_{21}		S_{12}		S_{22}		$\lvert S_{21}\rvert$(dB)
	MAG	ANG	MAG	ANG	MAG	ANG	MAG	ANG	
2.0	0.11	−99	1.98	159	0.02	95	0.14	−21	5.9
3.0	0.09	−148	2.05	124	0.03	57	0.29	−76	6.2
4.0	0.06	−146	2.12	94	0.04	27	0.29	−92	6.5
5.0	0.09	−120	2.16	64	0.04	−2	0.26	−95	6.7
6.0	0.13	−128	2.16	37	0.05	−30	0.25	−92	6.7
7.0	0.15	−145	2.16	9	0.06	−57	0.24	−91	6.7
8.0	0.13	−154	2.17	−17	0.07	−82	0.21	−93	6.7
9.0	0.10	−137	2.22	−45	0.08	−108	0.16	−93	6.9
10.0	0.19	−96	2.21	−73	0.09	−134	0.15	−61	6.9
11.0	0.10	−119	2.28	−102	0.10	−156	0.09	−51	7.1
12.0	0.14	−148	2.20	−129	0.11	−180	0.15	−30	6.9
13.0	0.11	−175	2.19	−157	0.11	155	0.20	−45	6.8
14.0	0.04	125	2.18	175	0.11	128	0.20	−69	6.8
15.0	0.07	−18	2.17	145	0.11	96	0.16	−94	6.7
16.0	0.13	−56	2.17	114	0.12	62	0.10	−118	6.7
17.0	0.11	−66	2.21	82	0.14	28	0.04	−82	6.9
18.0	0.19	−23	2.37	41	0.18	−7	0.18	−58	7.5

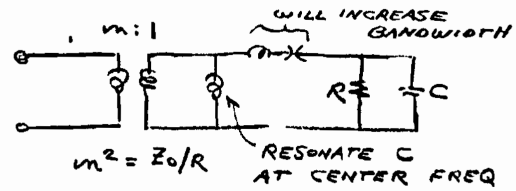
NOTE: Reference planes for S-parameter data located at center of device bond pads.

BANDWIDTH LIMIT FOR MATCHING A REACTIVE LOAD

A COMMON PROBLEM IN ELECTRONICS IS THE MATCHING OF A RESISTIVE GENERATOR TO A REACTIVE LOAD OVER A RANGE OF FREQUENCIES SAY f_L TO f_H



A SIMPLE NETWORK TO PERFORM THIS TASK IS SHOWN AT RIGHT. THERE IS NO FUNDAMENTAL PROBLEM IN THE RESISTIVE TRANSFORMATION R TO Z_0. AT LOW FREQUENCIES THIS CAN BE PERFORMED BY A TRANSFORMER AND AT HIGH FREQUENCIES BY A LONG TAPERED TRANSMISSION LINE. HOWEVER THE SIMPLE NETWORK PROVIDES Z_0 AT ONLY THE CENTER FREQUENCY



A FUNDAMENTAL BANDWIDTH LIMIT WAS DERRIVED BY FANO AND IS STATED IN TERMS OF THE REFLECTION COEFFICIENT $\Gamma = (Z_{IN} - Z_0)/(Z_{IN} + Z_0)$. FOR ANY LOSSLESS MATCHING NETWORK, NO MATTER HOW COMPLEX,

$$\int_0^{\infty} \ln \frac{1}{|\Gamma|}\, df \le \frac{1}{2RC} \qquad \text{FOR A PARALLEL RC LOAD}$$

REF: R.M. FANO, "THEORETICAL LIMITATIONS ON BROADBAND MATCHING OF ARBITRARY IMPEDANCES," J. OF FRANKLIN INST, JAN 1950, pp 57-154

(AND $\Gamma = 1$ AT ALL OTHER FREQUENCIES)

IF Γ IS A CONSTANT Γ_0 FROM f_L TO f_H THE INTEGRAL BECOMES

$$\ln \frac{1}{\Gamma_0} \cdot (f_H - f_L) \le \frac{1}{2RC} \qquad \text{OR} \qquad \Gamma_0 \ge e^{-\pi B/\Delta f} \qquad \text{WHERE } B \equiv 1/2\pi RC, \; \Delta f = f_H - f_L$$

I.E. IF $\Delta f = B$ (USUAL 3 DB BANDWIDTH OF SIMPLE RC)

THEN $\Gamma_0 = e^{-\pi} = .043 \qquad 20 \log \Gamma_0 = -27.3$ dB

SIMILAR RELATIONS EXIST FOR OTHER RC AND RL LOADS

SEE: G. GONZALEZ, MICROWAVE TRANSISTOR AMPLIFIERS, PRENTICE, 1984 pp 167-169
OR J. LEV, "CALCULATOR PROGRAM FINDS FANO BANDWIDTH," MICROWAVES AND RF SEP 1985, p. 153-155

SERIES RL

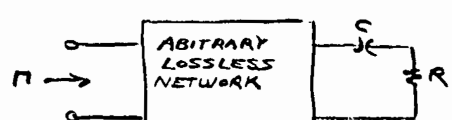
$$\int_0^{\infty} \ln \frac{1}{|\Gamma|}\, df \le \frac{R}{2L}$$

PARALLEL RL

$$\int_0^{\infty} \ln \frac{1}{|\Gamma|} \frac{df}{f^2} \le \frac{2\pi^2 L}{R}$$

SERIES RC

$$\int_0^{\infty} \ln \frac{1}{|\Gamma|} \frac{df}{f^2} \le 2\pi^2 RC$$



IF $\Gamma = \Gamma_0$ FROM f_L TO f_H AND 1 ELSEWHERE

$$\ln \frac{1}{\Gamma_0} \cdot \left(\frac{1}{f_L} - \frac{1}{f_H} \right) \le 2\pi^2 RC \qquad \ln \frac{1}{\Gamma_0} \le 2\pi^2 RC \cdot \frac{f_L f_H}{f_H - f_L} \approx \frac{\pi f_0^2}{B \cdot \Delta f}$$

IN THIS CASE THE BANDWIDTH, Δf INCREASES AS f_0^2 FOR A GIVEN RC

$f_0^2 = f_L f_H$

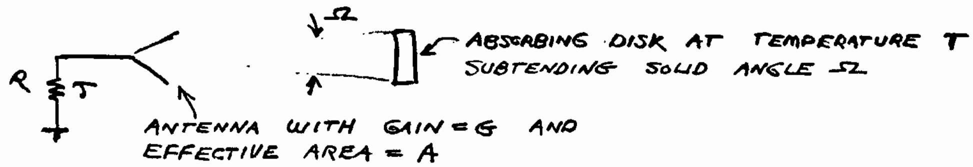
$B = (2\pi RC)^{-1}$

RELATION OF G TO A

AN ANTENNA IS RECIPROCAL AND ITS TRANSMITTING AND RECEIVING PROPERTIES ARE RELATED BY

$$G = 4\pi A/\lambda^2 \quad \text{OR} \quad A = G\lambda^2/4\pi$$

A SIMPLE THERMODYNAMIC PROOF IS AS FOLLOWS



THE THERMAL POWER TRANSMITTED FROM R TO DISK IS $P_T = G \cdot kTB \cdot \Omega / 4\pi$

THE DISK EMITS A FLUX $S = \frac{1}{2} \cdot 2kTB/\lambda^2 \cdot \Omega$ AT THE ANTENNA BY THE PLANCK RADIATION LAW (THE 1/2 ARISES BECAUSE THE ANTENNA RECEIVES 1/2 THE RANDOM POLARIZATION) $P_R = A \cdot S$

SINCE THE RESISTOR AND DISK ARE AT THE SAME T

$$P_R = P_T = A \cdot \frac{kTB}{\lambda^2} \cdot \Omega = \frac{G \cdot kTB \cdot \Omega}{4\pi} \quad \text{OR} \quad G = 4\pi A/\lambda^2$$

G, A, AND BW OF SOME COMMON ANTENNAS

TYPE	G	A	BW
ISOTROPIC	1	$\lambda^2/4\pi = 0.08\lambda^2$	360°
SHORT DIPOLE	1.5	$3\lambda^2/8\pi = .12\lambda^2$	90°
$\lambda/2$ DIPOLE	1.64	$.13\lambda^2$	78°
$\lambda/4$ MONOPOLE	3.28	$.26\lambda^2$	78°
PARABOLOID, DIAMETER = D PHYSICAL AREA = A_P	$4\pi A/\lambda^2$	$\sim 0.6 A_P$ $\sim 0.6 \pi D^2/4$	$\sim 1.3\lambda/D$

PATH LOSS EQUATIONS – FREE SPACE

USING EQUATIONS $S = P_T \cdot G_T/4\pi r^2$ AND $P_R = A_R \cdot S$ WE OBTAIN

$$\frac{P_R}{P_T} = \frac{G_T A_R}{4\pi r^2} = G_T G_R \cdot \left(\frac{\lambda}{4\pi r}\right)^2 = \frac{A_T A_R}{\lambda^2 r^2}$$

FOR TRANSMISSION BETWEEN TRANSMITTING ANTENNA (G_T OR A_T) AND RECEIVING ANTENNA (G_R OR A_R)

RADAR RANGE

THE REFLECTED POWER IS DESCRIBED BY THE RADAR CROSS SECTION σ WHICH IS DEFINED IN SUCH A WAY THAT THE REFLECTOR RECEIVES POWER $\sigma \cdot S$ AND REFLECTS ISOTROPICALLY ($G = \lambda^2/4\pi$) TO GIVE

$$\frac{P_R}{P_T} = \frac{G_T A_R \sigma}{(4\pi)^2 r^4} = \frac{G_T^2 \lambda^2 \sigma}{(4\pi)^3 r^4} = \frac{A_T^2 \sigma}{4\pi \lambda^2 r^4}$$

$$A_R = G_T \lambda^2/4\pi$$

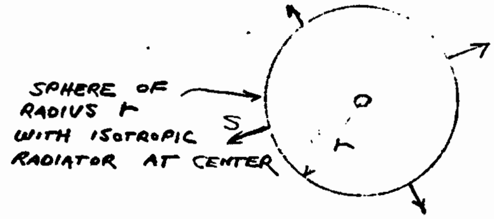
OBJECT	σ
SPHERE	πa^2 (RADIUS = a)
MAN	1 (ALL m²)
BIRD	.01
BOMBER	40
PARABOLOID	$4\pi A^2/\lambda^2$

ANTENNAS

CONSIDER AN ANTENNA AS A TRANSDUCER WHICH ACCEPTS TRANSMITTED POWER P_T AND CONVERTS THIS TO A POWER FLUX S (WATTS/m^2) AT A DISTANCE r FROM THE ANTENNA. IF THE ANTENNA IS AN ISOTROPIC RADIATOR THEN THIS POWER SPREADS OUT UNIFORMLY IN ALL DIRECTIONS AND BY CONSERVATION OF POWER THE FLUX IS

$$\text{FLUX} \equiv S = \frac{P_T}{4\pi r^2} \quad \frac{\text{WATTS}}{m^2}$$

FOR ISOTROPIC RADIATOR

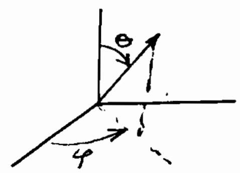


AN ANTENNA USUALLY DIRECTS THE POWER IN DESIRED DIRECTIONS AS DESCRIBED BY A GAIN FUNCTION $G(\theta,\varphi)$ WHICH IS JUST THE RATIO OF FLUX IN DIRECTION θ,φ TO THAT OF AN ISOTROPIC RADIATOR. THUS

$$S = \frac{P_T\, G}{4\pi r^2} \quad \text{FOR ANTENNA WITH GAIN } G$$

BY CONSERVATION OF POWER THE SURFACE INTEGRAL OF S AROUND A SPHERE OF ANY RADIUS MUST EQUAL P_T

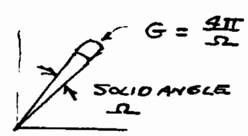
$$\int S\, r^2 \sin\theta\, d\theta\, d\varphi = P_T \Rightarrow \int G \sin\theta\, d\theta\, d\varphi = 4\pi$$



THE GAIN INTEGRATED OVER ALL ANGLES IS 4π

IF ALL POWER FROM AN ANTENNA IS CONCENTRATED IN A BEAM OF SOLID ANGLE Ω THEN

$$G = \frac{4\pi}{\Omega} \quad \text{FOR } \theta,\varphi \text{ WITHIN } \Omega \qquad G = 0 \quad \text{ELSEWHERE}$$



FOR A CIRCULAR BEAM OF BEAM WIDTH BW (RADIANS) THE SOLID ANGLE $\Omega = \frac{\pi (BW)^2}{4}$ AND $G = 16/(BW)^2$

EXAMPLE: FROM A COMMUNICATION SATELLITE IN SYNCHRONOUS ORBIT (22,000 MILE HEIGHT) THE U.S. SUBTENDS A SOLID ANGLE OF

$$\Omega_{U.S.} = \frac{2500}{22000} \times \frac{1500}{22000} = .0077 \text{ STEARADIANS}$$

AN ANTENNA COVERING THE U.S. WOULD HAVE $G = 4\pi/.0077 = 1632$ OR 32.1 DB

EFFECTIVE AREA, A

AS A RECEIVING TRANSDUCER AN ANTENNA CONVERTS AN INCIDENT FLUX S INTO RECEIVED POWER P_R BY

$$P_R = S \cdot A$$

WHERE A IS THE EFFECTIVE AREA OF THE ANTENNA.

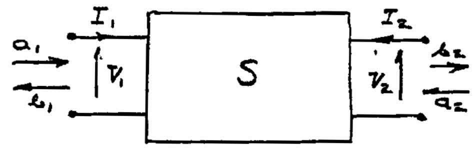
FOR AN EFFICIENT PARABOLIC REFLECTOR $A \sim 0.6 A_p$ WHERE $A_p = \pi D^2/4$ IS THE PROJECTED PHYSICAL AREA OF A PARABOLOID OF DIAMETER D

REFERENCES: REF DATA FOR ENGINEERS, E.C. JORDAN, SAMS 1985, CHAPTER 32

ANTENNAS, J. D. KRAUS, McGRAW HILL, 1950

S
PARAMETER
SUMMARY
S. WEINREB 4/78

SCATTERING, S, PARAMETERS



$$a = \frac{V + I Z_0}{2 Z_0^{1/2}}$$

$$b = \frac{V - I Z_0}{2 Z_0^{1/2}}$$

$$V = (a + b) Z_0^{1/2}$$

$$I = (a - b) Z_0^{-1/2}$$

MATRIX FORM

$$b] = [S]\, a]$$

TWO-PORT EQUATIONS

$$b_1 = S_{11} a_1 + S_{12} a_2$$

$$b_2 = S_{21} a_1 + S_{22} a_2$$

RECIPROCITY

$$[S] = [S^T] \qquad S_{21} = S_{12}$$

LOSSLESSNESS

$$[S^*][S^T] = [1]$$

$$(\text{Row})_i \times (\text{Row})_j = \delta_{ij}$$

$$|S_{11}|^2 + |S_{12}|^2 = 1$$

$$|S_{21}|^2 + |S_{22}|^2 = 1$$

$$S_{11} S_{21}^* + S_{12} S_{22}^* = 0$$

IF LOSSLESS AND RECIPROCAL

$$|S_{22}| = |S_{11}| \qquad |S_{12}| = |S_{21}| = \sqrt{1 - |S_{11}|^2}$$

$$\phi_{12} = \phi_{21} = \frac{\phi_{11} + \phi_{22}}{2} + \frac{\pi}{2} \pm n\pi$$

IN TERMS OF Z

$$[S] = [Z - 1][Z + 1]^{-1}$$

$$D S_{11} = (z_{11} - 1)(z_{22} + 1) - z_{12} z_{21}$$

$$D S_{12} = 2 z_{12} \qquad D S_{21} = 2 z_{21}$$

$$D S_{22} = (z_{22} - 1)(z_{11} + 1) - z_{12} z_{21}$$

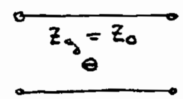
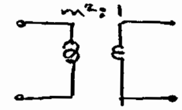
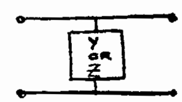
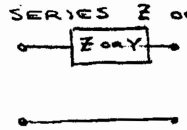
$$D = (z_{11} + 1)(z_{22} + 1) - z_{12} z_{21}$$

IN TERMS OF T (CLASS DEFINITION)

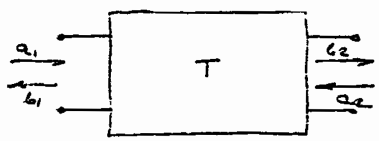
$$S_{11} = T_{21}/T_{11} \qquad S_{12} = (T_{22} T_{11} - T_{12} T_{21})/T_{11}$$

$$S_{21} = 1/T_{11} \qquad S_{22} = -T_{12}/T_{11}$$

S PARAMETERS FOR BASIC TWO-PORTS

DEVICE	PARAMETERS	
T-LINE  $Z_g = Z_0$, θ	$\begin{bmatrix} 0 & e^{-j\theta} \\ e^{-j\theta} & 0 \end{bmatrix}$	$\theta = \omega T = \beta l = \frac{2\pi l}{\lambda_g}$
IDEAL TRANSFORMER  $m^2:1$	$\begin{bmatrix} \frac{m^2-1}{m^2+1} & \frac{2m^{-1}}{m^{-2}+1} \\ \frac{2m}{m^2+1} & \frac{m^{-2}-1}{m^{-2}+1} \end{bmatrix}$	m = TURNS RATIO
SHUNT Y OR Z 	$\begin{bmatrix} \frac{-Y}{Y+2} & \frac{2}{Y+2} \\ \frac{2}{Y+2} & \frac{-Y}{Y+2} \end{bmatrix}$ OR $\begin{bmatrix} \frac{-1}{2Z+1} & \frac{2Z}{2Z+1} \\ \frac{2Z}{2Z+1} & \frac{-1}{2Z+1} \end{bmatrix}$	
SERIES Z OR Y 	$\begin{bmatrix} \frac{Z}{Z+2} & \frac{2}{Z+2} \\ \frac{2}{Z+2} & \frac{Z}{Z+2} \end{bmatrix}$ OR $\begin{bmatrix} \frac{1}{2Y+1} & \frac{2Y}{2Y+1} \\ \frac{2Y}{2Y+1} & \frac{1}{2Y+1} \end{bmatrix}$	

T - PARAMETER SUMMARY



$$a_1 = T_{11} b_2 + T_{12} a_2$$
$$b_1 = T_{21} b_2 + T_{22} a_2$$

RECIPROCITY: $T_{22} T_{11} - T_{21} T_{12} = 1$

LOSSLESS:
$$|T_{11}|^2 - |T_{21}|^2 = 1$$
$$|T_{22}|^2 - |T_{12}|^2 = 1$$
$$T_{11} T_{12}^* - T_{22} T_{21}^* = 0$$
OR
$$|T_{22}| = |T_{11}|$$
$$|T_{21}| = |T_{12}| = \sqrt{|T_{11}|^2 - 1}$$
$$\phi_{11} - \phi_{12} = \phi_{22} - \phi_{21}$$

RECIPROCAL AND LOSSLESS:
$$T_{22} = T_{11}^* \qquad \phi_{22} = -\phi_{11} \overset{?}{=} \phi_{21} = -\phi_{12}$$
$$T_{21} = T_{12}^*$$

RELATION TO S

$T_{11} = 1/S_{21}$ $\quad T_{12} = -S_{22}/S_{21}$ $\quad S_{11} = T_{21}/T_{11}$

$T_{21} = S_{11}/S_{21}$ $\quad T_{22} = \dfrac{S_{21}S_{12} - S_{11}S_{22}}{S_{21}}$ $\quad S_{21} = 1/T_{11}$

$S_{12} = (T_{11}T_{22} - T_{12}T_{21})/T_{11}$

$S_{22} = -T_{12}/T_{11}$

RELATION TO ABCD

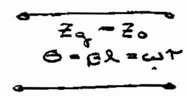
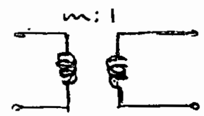
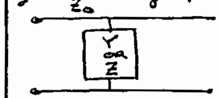
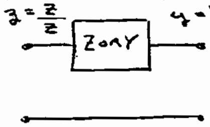
$T_{11} = 1/2\,(A + BZ_0^{-1} + CZ_0 + D)$ $\quad T_{12} = 1/2\,(A - BZ_0^{-1} + CZ_0 - D)$

$T_{21} = 1/2\,(A + BZ_0^{-1} - CZ_0 - D)$ $\quad T_{22} = 1/2\,(A - BZ_0^{-1} - CZ_0 + D)$

$A = 1/2\,(T_{11} + T_{12} + T_{21} + T_{22})$ $\quad B = Z_0/2\,(T_{11} - T_{12} + T_{21} - T_{22})$

$C = (1/2Z_0)(T_{11} + T_{12} - T_{21} - T_{22})$ $\quad D = 1/2\,(T_{11} - T_{12} - T_{21} + T_{22})$

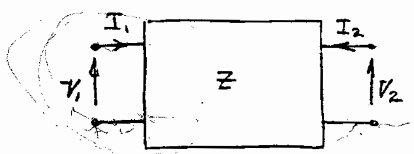
BASIC TWO PORTS FOR T PARAMETERS

 $Z_g = Z_0$, $\theta = \beta l = \omega\tau$	$e^{+\gamma\theta}$	0			
	0	$e^{-\gamma\theta}$			
 $m:1$	$\frac{m^2+1}{2m}$	$\frac{m^2-1}{2m}$			
	$\frac{m^2-1}{2m}$	$\frac{m^2+1}{2m}$			
 $z = \frac{Z}{Z_0}$, $y = YZ_0$	$1 + 1/2z$	$1/2z$	OR	$1 + y/2$	$y/2$
	$-1/2z$	$1 - 1/2z$		$-y/2$	$1 - y/2$
 $z = \frac{Z}{Z}$, $y = YZ_0$	$1 + z/2$	$-z/2$	OR	$1 + 1/2y$	$-1/2y$
	$z/2$	$1 - z/2$		$1/2y$	$1 - 1/2y$

INVERSE
$$a_2 = (T_{11} b_1 - T_{21} a_1)/E$$
$$b_2 = (-T_{12} b_1 + T_{22} a_1)/E$$
$$E = T_{11}T_{22} - T_{21}T_{12} \qquad E = 1 \text{ IF RECIPROCAL}$$

SYMMETRY $\quad T_{12} = -T_{21}$; $\quad T_{21} = -T_{12}$ AND $E = 1$

IMPEDENCE, Z, PARAMETERS – SUMMARY



MATRIX FORM	TWO-PORT EQUATIONS
$V] = [Z]\ I]$	$V_1 = Z_{11}I_1 + Z_{12}I_2$ $V_2 = Z_{21}I_1 + Z_{22}I_2$

RECIPROCITY

$[Z] = [Z]^T$	$Z_{12} = Z_{21}$

LOSSLESSNESS

$[Z] = -[Z]^{\dagger}$	$\mathrm{Re}\, Z_{11} = \mathrm{Re}\, Z_{22} = 0$ $\mathrm{Re}\, Z_{12} = -\mathrm{Re}\, Z_{21}$ $\mathrm{Im}\, Z_{12} = \mathrm{Im}\, Z_{21}$ NO

LOSSLESS AND RECIPROCAL

$\mathrm{Re}\,[Z] = 0$	$\mathrm{Re}\, Z_{11} = \mathrm{Re}\, Z_{22} = 0$ $\mathrm{Re}\, Z_{12} = \mathrm{Re}\, Z_{21} = 0$ $\mathrm{Im}\, Z_{12} = \mathrm{Im}\, Z_{21}$

IN TERMS OF S PARAMETERS

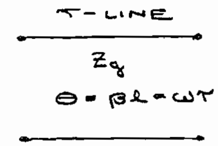
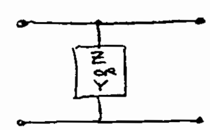
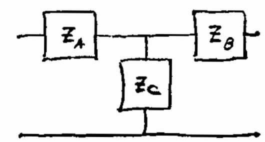
$[Z] = [1+S][1-S]^{-1} Z_0$	$D\, Z_{11} = (1+S_{11})(1-S_{22}) + S_{12}S_{21}$ $D\, Z_{12} = 2S_{12}$ $D\, Z_{21} = 2S_{21}$ $D\, Z_{22} = (1+S_{22})(1-S_{11}) + S_{12}S_{21}$ $D = [(1-S_{11})(1-S_{22}) - S_{12}S_{21}]/Z_0$

IN TERMS OF ABCD PARAMETERS

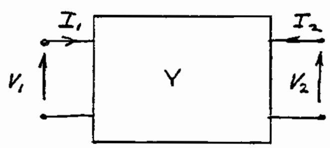
$A = Z_{11}/Z_{21}$ $\quad B = \dfrac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}$ $\qquad Z_{11} = A/C$ $\quad Z_{12} = \dfrac{AD - BC}{C}$

$C = 1/Z_{21}$ $\quad D = Z_{22}/Z_{21}$ $\qquad Z_{21} = 1/C$ $\quad Z_{22} = D/C$

Z PARAMETERS FOR BASIC TWO-PORTS

	$\begin{bmatrix} -jZ_g\cot\theta & -jZ_g/\sin\theta \\ -jZ_g/\sin\theta & -jZ_g\cot\theta \end{bmatrix}$
	$\begin{bmatrix} Z & Z \\ Z & Z \end{bmatrix}$ $\quad Z = 1/Y$
	$Z_A + Z_C \quad Z_C$ $Z_C \quad Z_B + Z_C$

Y – PARAMETER SUMMARY



$[I] = [Y][V]$

$I_1 = Y_{11}V_1 + Y_{12}V_2$

$I_2 = Y_{21}V_1 + Y_{22}V_2$

RECIPROCITY

$[Y] = [Y]^T$ $\qquad Y_{12} = Y_{21}$

LOSSLESS

$[Y] = -[Y]^{+}$

$Re\,Y_{11} = Re\,Y_{22} = 0$
$Re\,Y_{12} = -Re\,Y_{21}$
$Im\,Y_{12} = Im\,Y_{21}$

LOSSLESS AND RECIPROCAL

$Re\,[Y] = 0$

$Re\,Y_{11} = Re\,Y_{22} = 0$
$Re\,Y_{12} = Re\,Y_{21} = 0$
$Im\,Y_{12} = Im\,Y_{21}$

RELATION TO S

$Z_0[Y] = [1-S][1+S]^{-1}$

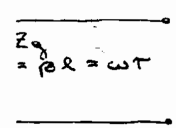
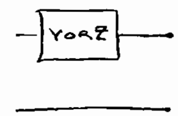
$D\,Y_{11} = (1-S_{11})(1+S_{22}) + S_{12}S_{21}$
$D\,Y_{12} = -2S_{12}$
$D\,Y_{21} = -2S_{21}$
$D\,Y_{22} = (1-S_{22})(1+S_{11}) + S_{12}S_{21}$
$D = Z_0[(1+S_{11})(1+S_{22}) - S_{12}S_{21}]$

RELATION TO ABCD (ERROR IN ALTMAN)

$Y_{11} = D/B \qquad Y_{12} = -(AD-BC)/(B)$

$Y_{21} = -1/B \qquad Y_{22} = A/B$

BASIC PARAMETERS – Y PARAMETERS

 Z_g $= \beta\ell = \omega T$	$-\dfrac{j\cot\theta}{Z_g}$ $\quad \dfrac{j}{Z_g\sin\theta}$	
	$\dfrac{j}{Z_g\sin\theta}$ $\quad -\dfrac{j\cot\theta}{Z_g}$	
 Y OR Z	$Y \quad -Y$	OR $\quad Z^{-1} \quad -Z^{-1}$
	$-Y \quad Y$	$-Z^{-1} \quad Z^{-1}$

INVERSE – RELATION TO Z

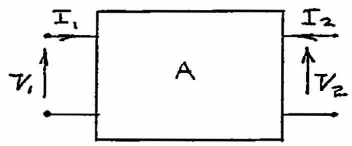
$[Y] = [Z]^{-1}$

$D\,Y_{11} = Z_{22} \qquad D\,Y_{12} = -Z_{12}$

$D\,Y_{21} = -Z_{12} \qquad D\,Y_{22} = Z_{11}$

$D = Z_{11}Z_{22} - Z_{12}Z_{21}$

ABCD PARAMETER SUMMARY



$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

RECIPROCITY $\quad AD - BC = 1$

LOSSLESSNESS

$\mathrm{Re}\, AC^* = 0 \qquad \mathrm{Re}\, B/A = 0$

$\mathrm{Re}\, BD^* = 0 \qquad \mathrm{Re}\, D/A = 0$

$\mathrm{Im}\, AD^* = \mathrm{Im}\, BC^* \qquad \mathrm{Re}\, A^*B = \mathrm{Re}\, C^*D$

RECIPROCAL AND LOSSLESS

$\mathrm{Im}\, A = \mathrm{Im}\, D = 0$

$\mathrm{Re}\, B = \mathrm{Re}\, C = 0$

RELATION TO Z

$A = Z_{11}/Z_{21} \qquad B = (Z_{11}Z_{22} - Z_{12}Z_{21})/Z_{21}$

$C = 1/Z_{21} \qquad D = Z_{22}/Z_{21}$

$Z_{11} = A/C \qquad Z_{12} = (AD - BC)/C$

$Z_{21} = 1/C \qquad Z_{22} = D/C$

RELATION TO S

$$A = \frac{S_{12}S_{21} + (1+S_{11})(1-S_{22})}{2S_{21}} \qquad B = \frac{Z_0(-S_{12}S_{21} + (1+S_{11})(1+S_{22}))}{2S_{21}}$$

$$C = \frac{-S_{12}S_{21} + (1-S_{11})(1-S_{22})}{2Z_0S_{21}} \qquad D = \frac{S_{12}S_{21} + (1-S_{11})(1+S_{22})}{2S_{21}}$$

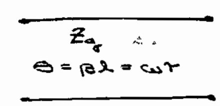
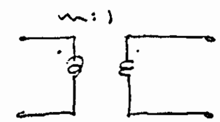
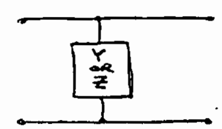
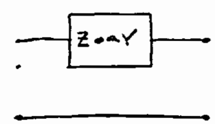
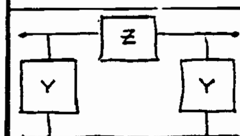
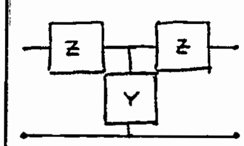
$ES_{11} = A + B/Z_0 - CZ_0 - D \qquad ES_{12} = 2(AD - BC)$

$ES_{21} = 2 \qquad ES_{22} = -A + B/Z_0 - CZ_0 + D$

$E = A + B/Z_0 + CZ_0 + D$

RELATION TO T (ON T SUMMARY)

BASIC TWO-PORT ABCD PARAMETERS

	$\begin{matrix} \cos\theta & jZ_0\sin\theta \\ jZ_0^{-1}\sin\theta & \cos\theta \end{matrix}$
	$\begin{matrix} n & 0 \\ 0 & n^{-1} \end{matrix}$
	$\begin{matrix} 1 & 0 \\ Y & 1 \end{matrix}$ or $\begin{matrix} 1 & 0 \\ Z^{-1} & 1 \end{matrix}$
	$\begin{matrix} 1 & Z \\ 0 & 1 \end{matrix}$ or $\begin{matrix} 1 & Y^{-1} \\ 0 & 1 \end{matrix}$
	$\begin{matrix} 1+YZ & Z \\ Y(2+YZ) & 1+YZ \end{matrix}$
	$\begin{matrix} 1+YZ & Z(2+YZ) \\ Y & 1+YZ \end{matrix}$

INVERSE $\quad V_2 = (DV_1 - BI_1)/E$

$I_2 = (CV_1 - AI_1)/E$

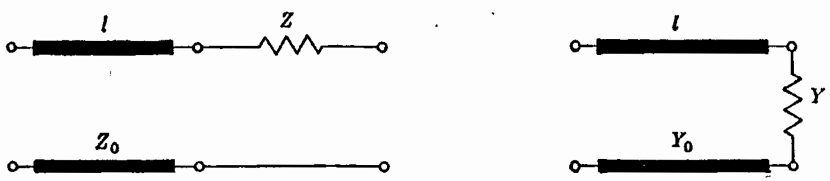
$E = AD - BC \qquad E = 1$ IF RECIPROCAL

SYMMETRY $\quad A = D$ AND $E = 1$

APPENDIX B – LOSSLESS TWO-PORT MODELS

(FROM M.I.T. RAD LAB, VOL 8, PRINC. OF MIC. CIRCUITS MONTGOMERY, DICKE, AND PURCELL) p. 105

Z_0, l, Z MODEL



(a) (b)

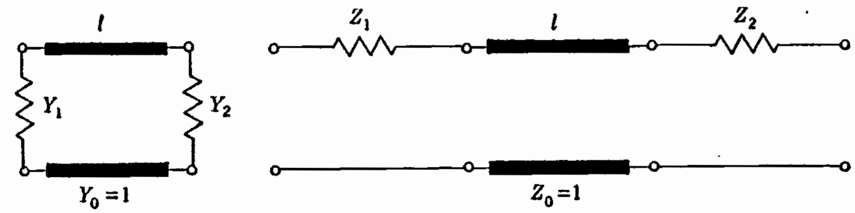
FIG. 4·16.

$$Z_{11} = -jZ_0 \cot \beta l, \qquad Z = Z_{22} - Z_{11},$$

$$Z_{22} = Z - jZ_0 \cot \beta l, \qquad \beta l = \cos^{-1} \frac{Z_{11}}{Z_{12}},$$

$$Z_{12} = \pm jZ_0 \csc \beta l, \qquad Z_0 = jZ_{12} \sqrt{1 - \left(\frac{Z_{11}}{Z_{12}}\right)^2}.$$

Y_1, l, Y_2 MODEL



(a) (b)

FIG. 4·17.

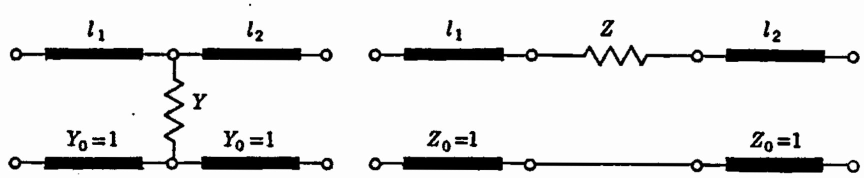
$$Y_{11} = Y_1 - j \cot \beta l, \qquad Y_1 = Y_{11} \mp \sqrt{1 + Y_{12}^2}$$

$$Y_{22} = Y_2 - j \cot \beta l, \qquad Y_2 = Y_{22} \mp \sqrt{1 + Y_{12}^2}$$

$$Y_{12} = \pm j \csc \beta l, \qquad \beta l = \csc^{-1}(\mp jY_{12}) = \sin^{-1}\left(\frac{\pm j}{Y_{12}}\right).$$

(CORRECTED FROM MONTGOMERY, ET AL)

l_1, Y, l_2 MODEL



(a) (b)

FIG. 4·19.

$$Y_{11} = j\frac{1 - \cot \beta l_1 \cot \beta l_2 - jY \cot \beta l_1}{\cot \beta l_1 + \cot \beta l_2 + jY},$$

$$Y_{22} = j\frac{1 - \cot \beta l_1 \cot \beta l_2 - jY \cot \beta l_2}{\cot \beta l_1 + \cot \beta l_2 + jY},$$

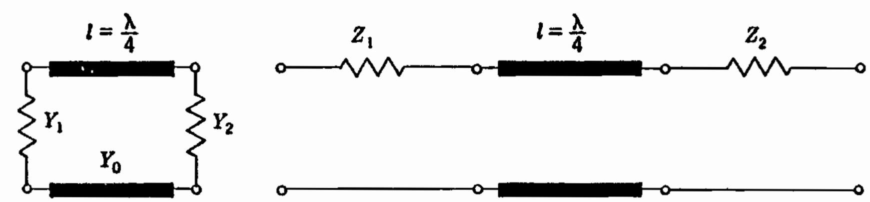
$$Y_{12} = \frac{j \csc \beta l_1 \csc \beta l_2}{\cot \beta l_1 + \cot \beta l_2 + jY}.$$

Symmetrical Case Only

$$Y = \frac{1}{Y_{12}[1 - (Y_{11} + Y_{12})^2]} + 2(Y_{11} + Y_{12}),$$

$$\beta l = \cot^{-1}(-jY_{11} - jY_{12}).$$

Y_1, Y_0, Y_2 MODEL



(a) (b)

FIG. 4·18.

$$Y_{11} = Y_1, \qquad Y_1 = Y_{11},$$

$$Y_{22} = Y_2, \qquad Y_2 = Y_{22},$$

$$Y_{12} = jY_0, \qquad Y_0 = -jY_{12}.$$

NOTE:
1) Z_0 OR Y_0 = 1 MEANS EQUATIONS ARE NORMALIZED TO Z_0 OR Y_0
2) FOR RIGHT-HAND (b) CIRCUIT SUBSTITUTE Y FOR Z OR Z FOR Y IN EQUATIONS
3) —WW— MEANS REACTANCE

LOSSLESS TWO-PORT MODELS

l, Z₀, n MODEL

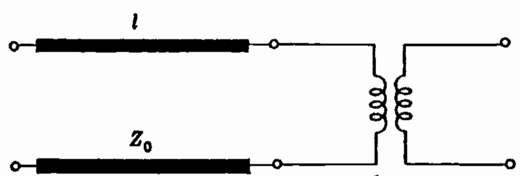


FIG. 4·20.

$$Z_{11} = -jZ_0 \cot \beta l, \qquad \beta l = \cos^{-1} \frac{\sqrt{Z_{11}Z_{22}}}{Z_{12}},$$

$$Z_{22} = -jn^2 Z_0 \cot \beta l, \qquad Z_0 = -jZ_{11}\sqrt{\frac{Z_{12}^2}{Z_{11}Z_{22}} - 1},$$

$$Z_{12} = jnZ_0 \csc \beta l, \qquad n = \sqrt{\frac{Z_{22}}{Z_{11}}}.$$

l_1, n, l_2 MODEL

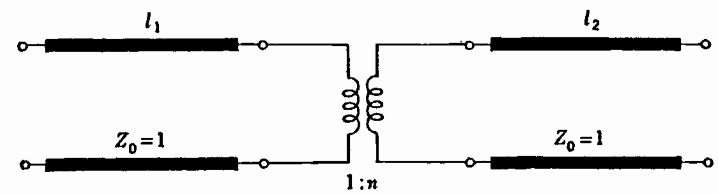


FIG. 4·21.

$$Z_{11} = j \tan [\beta l_1 + \tan^{-1} (n^2 \tan \beta l_2)],$$

$$Z_{22} = j \tan \left[\beta l_2 + \tan^{-1}\left(\frac{\tan \beta l_1}{n^2}\right)\right],$$

$$Z_{11}Z_{22} - Z_{12}^2 = \frac{n^2 \tan \beta l_1 \tan \beta l_2 - 1}{\tan \beta l_1 \tan \beta l_2 - n^2},$$

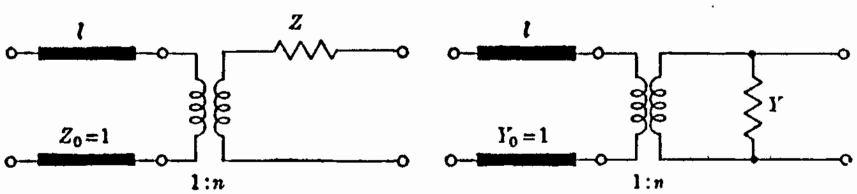
$$\tan \beta l_1 = \frac{1 + c^2 - a^2 - b^2}{2(bc - a)} \pm \sqrt{\left[\frac{1 + c^2 - a^2 - b^2}{2(bc - a)}\right]^2 + 1},$$

$$\tan \beta l_2 = \frac{b + c\alpha}{-\alpha + a} = \frac{1 + \alpha a}{c - \alpha b},$$

$$n^2 = \frac{-c\alpha - b}{1 + \alpha a} = -\frac{a - \alpha}{c - \alpha b},$$

$$a = -jZ_{11},$$
$$b = Z_{11}Z_{22} - Z_{12}^2,$$
$$c = -jZ_{22},$$
$$\alpha = \tan \beta l_1.$$

l, n, Z MODEL



(a) (b)

FIG. 4·22.

$$Z_{11} = -j\cot \beta l, \qquad \beta l = \cot^{-1} jZ_{11},$$

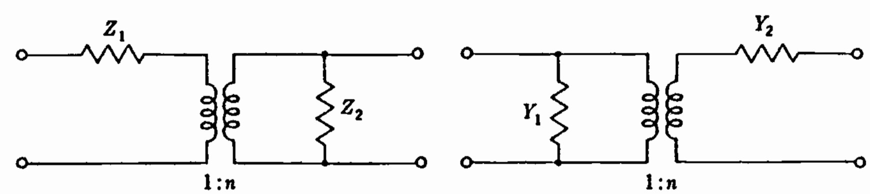
$$Z_{22} = Z - jn^2 \cot \beta l, \qquad n = \frac{\pm j\, Z_{12}}{\sqrt{1 - Z_{11}^2}},$$

$$Z_{12} = -jn\sqrt{\cot^2 \beta l + 1}, \qquad Z = Z_{22} + \frac{Z_{11}Z_{12}^2}{1 - Z_{11}^2}.$$

$$= -jn/\sin \beta l$$

(CORRECTED FROM REFERENCE)

Z_1, n, Z_2 MODEL



(a) (b)

FIG. 4·23.

$$Z_{11} = Z_1 + \frac{Z_2}{n^2}, \qquad Z_1 = Z_{11} - \frac{Z_{12}^2}{Z_{22}},$$

$$Z_{22} = Z_2, \qquad Z_2 = Z_{22},$$

$$Z_{12} = \pm \frac{Z_2}{n}. \qquad n = \frac{Z_{22}}{Z_{12}}.$$